

MAPS OF CONTINUOUS SPATIAL DEPENDENCE

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Abstract – *Heterogeneity is one of the distinguishing features in spatial econometric models. It is a frequent problem in applied work and can be very damaging for statistical inference. In this paper, we focus on the problems implied by the existence of instabilities in the mechanism of spatial dependence in a spatial lag model, assuming that the other terms of the specification remain stable. We begin the discussion with the role played by the algorithms of local estimation in detecting the instabilities. Problems appear when one must decide what to do once the existence of heterogeneity has been confirmed. The logical reaction is trying to parameterize this lack of stability. However, the solution is not obvious. Assuming that a set of indicators related to the problem has been identified, we propose a simple technique to deal with the unknown functional form. In the final part of the paper, we present some Monte Carlo evidence and an application to evaluate the instability in the mechanisms of spatial dependence in the convergence process of the European Regions.*

Keywords: DEPENDENCE, LOCAL ESTIMATION, MONTE-CARLO, SPATIAL INSTABILITY.

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1. INTRODUCTION

Spatial econometric models are very often affected by problems caused by the lack of constancy of some of their elements. There are many reasons explaining the absence of stability in a given model. On the one hand, it is possible that instability arises from a chain of purely random shocks affecting the behaviour of the model across space. In this situation, the main problem is testing for the hypothesis of overall stability. On the other hand, the symptoms of heterogeneity may follow some regular pattern that can be known, to some extent, by the user. Obviously, the problem can be due to a wrong selection of the functional form, which causes anomalies in the estimation, including outliers. This function may not be the same across space or it may change according to some specific factor. The omission of important variables from the specification, whose impact varies from place to place, is another cause of concern. Additionally, the parameters of the model may evolve across space.

In any of these situations, it would be difficult to maintain the original specification. The consequences are easily predictable: if we ignore the lack of stability, or if the solution adopted is not appropriate, the estimations are biased and inconsistent and the inference is misleading.

Mur et al. (2009a) advance in this discussion by means of a battery of tests, the purpose of which is to test for the null hypothesis of stability in, respectively, (i) the systematic part of the equation, (ii) the mechanisms of spatial dependence, (iii) the residuals of the model and (iv) any combination of the previous elements. Furthermore, Angulo et al. (2008) show that the existence of instability in some of these elements may produce false symptoms of instability in the others. This is the reason for developing a new battery of robust stability tests, with the aim of identifying the origin of the heterogeneity.

Our paper focuses on the particular case of the lack of stability in the mechanisms of spatial dependence, assuming that the user has some prior information on its causes. Specifically, we assume that heterogeneity follows some spatial pattern. This framework can be generalized by introducing exogenous variables associated with the problem of instability (e.g Farber et al., 2008, point to the topological features of the network, to the number of connections of each node, as the potential source of instability).

The simplest case of heterogeneity corresponds to the existence of a binary regime in the parameter of spatial interaction, where this parameter may take one of two values depending on the location of each point. The final result is very similar to the model of spatial regimes suggested by Anselin (1990), where there are a finite number of breaks. This discussion is not new in the literature. In fact, we may find a very interesting collection of papers that address a similar problem: Rietveld and Wintershoven (1998), Brunson et al. (1998b), Leung et al. (2000, 2003), Pace and Lesage (2004), LaCombe (2004) where a formal test of instability ‘*by regimes*’ appears. There are also different applications, among which we may cite the works of McMillen and McDonald (1996), Páez et al. (2002a and b), McMillen (2004).

More specifically, there are many papers in the literature analyzing the instability of the parameters that model the convergence process in the European regions. The papers by Ertur et al. (2006), Le Gallo and Dall'erba (2006), Fisher and Stirböck (2006) or Ramajo et al. (2008) are some of the most recent appearances on the subject. In general, all of them use dummy variables with the aim of establishing differences among the European regions in the convergence rates, in what has come to be known as *convergence clubs*. Following the same line we can find the papers of Mur et al. (2008, 2009b) that extend the instability analysis not only in the parameters of the exogenous part of the equation, but also in the dependence structure of the European Regions. There are enough reasons that justify the introduction of instability in the spatial dependence process as stated by Mur et al. (2008): "*it seems unrealistic to assume that the Eastern regions, for example, should maintain relations with the rest of the territory of similar intensity to those of their Western counterparts. In other words, if the distribution of infrastructures in space is very uneven (especially those that have to do with communications between agents), it is reasonable to suppose that the capacity for interrelations with neighbors should also suffer*".

We are interested in the case of a spatial continuous break corresponding to a situation where the parameter of spatial dependence may change at each point in space. Specifically, our paper focuses on the problem of solving the estimation of the instability mechanisms that intervene in a given model, where the break is of a continuous type.

The paper consists of six sections. In the second section, we review some fundamental concepts that underline the discussion about the lack of uniformity in the mechanisms of spatial dependence. In the third section, we develop a simple technique to obtain a preliminary estimation of the problem of instability and complete the discussion with a battery of specification tests. The fourth section contains a Monte Carlo study directed at checking the behaviour of these techniques. In the fifth section, we apply our proposals to two cases taken from the literature of applied spatial econometrics. We finish the paper with a brief section of conclusions and future prospects on the topic.

2. SOME ISSUES IN THE TREATMENT OF LOCAL INSTABILITIES IN THE MECHANISMS OF SPATIAL DEPENDENCE

There is not a lot of experience in handling models with problems of instability in the mechanisms of spatial dependence. The most important references, from our point of view, have already been cited in the previous section. Probably, the complexity of the algorithms of local estimation in non-linear models is a factor that has delayed its development. Nevertheless, it must be acknowledged that the problem is tangible (it is another, more compact way of looking at the question of the LISA treated as singularities in the structure of spatial dependence, Anselin, 1995), important from a theoretical point of view (as shown by López-Bazo et al., 2004; Parent and Riou, 2005; Ertur and Koch, 2007; Parent and Lesage, 2008) and with serious econometric consequences. We expect this subject to grow in importance in the near future.

To motivate the discussion, there are a series of fundamental aspects to which we would like to give briefly our attention. They are the following:

- (i) Testing vs Modelling the instability
- (ii) Continuous vs Discrete instability patterns
- (iii) Informative vs Non-informative estimation algorithms
- (iv) The bandwidth.

Testing should, logically, precede modelling in order to indicate how the latter should be carried out. However, this joint approach has not been usual. As mentioned before, we can find a wide range of tests of instability directed at the different elements of the model, taken individually or in blocks (Anselin, 1988b). Nevertheless, the question of the modelling is still in an embryonic state, even though the initial research into the subject of instability focused on the problem of the estimation (Casetti, 1972; McMillen, 1996; Brunson et al., 1998a).

The second question deals with the characterisation of the break, whether it is discrete or continuous. The first assumption (discrete break) is relatively popular in the applied literature, where the concept of the spatial regime (in reference to a model in which various structures of parameters coexist) is commonly used (Fisher and Stirböck, 2006, for example). Generally, the discussion is limited to the regression coefficients or to the variance, using an arbitrary division of space. The continuous approach is based on the concept of the hypersurface of parameters, introduced by the Geographically Weighted Regression literature (GWR in the following; e.g., Fotheringham et al., 1998). The latter technique focuses on the treatment of instability in the regression coefficients of spatial static models and, as is well known, produces biased estimators. Nevertheless, the bias of the GWR estimators will be, at worst, less than or equal to the Least Squares (LS in what follows), which do not contemplate, at all, the problem of instability.

The contribution of our paper lies in the distinction between informative and non-informative approaches with respect to the type of break that affects the mechanisms of spatial interaction. Mur et al. (2009a) and Angulo et al. (2008) associate the break to the existence of certain indicators that play an active role in the creation of instability. As examples, we can cite the cases of communications infrastructures in space, regional endowments of human capital or the position of the *nodes* in networks of spatial interaction. This kind of *a priori* information is very valuable, and it must be used to model the instability as well as to obtain heterogeneity tests with better properties than those based on a discrete approach. Our proposal is to progress towards a more compact framework that combines the two aspects: first, it is necessary to detect and to characterise the break; then, this information should be reintroduced into the model to solve the specification adequately.

The following example shows that, indeed, the information produced by the algorithms of local estimation is really useful. We take the case of a spatial

lag model (SLM in what follows) with a structure of instability in the parameter of spatial dependence:

$$\begin{cases} y = \rho \mathbf{H} \mathbf{W} y + X \beta + \varepsilon \\ \varepsilon \sim \mathbf{N}(0, \sigma^2 I) \end{cases} \quad (1)$$

$$\mathbf{H} = \text{diag} \{h_r; r = 1, 2, \dots, R\}; y = \begin{matrix} y_1 \\ y_2 \\ \dots \\ y_R \end{matrix}_{R \times 1}; X = \begin{matrix} 1 & x_1 \\ 1 & x_2 \\ \dots & \dots \\ 1 & x_R \end{matrix}_{R \times 2}; \beta = \begin{matrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_2 \end{matrix}_{2 \times 1}$$

This could be the case of a convergence model among European regions where externalities ($\mathbf{W}y$) with different intensities (h_r) for different regions are introduced. In this case, equation (1) can be written:

$$g_{y_{rt,t+n}} = \alpha + \beta \ln y_{0rt} + \rho h_r \sum_{s=1}^R w_{rs} g_{y_{st,t+n}} + \varepsilon_{rt}$$

where y_{rt} is the regional per capita income in region r and year t ; $g_{y_{rt,t+n}}$ is the corresponding growth rate between the years t and $t+n$; $\ln y_{0rt}$ is the logarithm of the per capita income in region r and the base year t ; w_{rs} refers to the (r,s) element of the spatial \mathbf{W} weighting matrix.

We continue the discussion from a theoretical point of view. Suppose that in the case of a square regular lattice, the problem of instability follows a well-defined spatial structure, for example, an elliptical paraboloid:

$$h_r(\alpha, a, b; c_1, c_2) = \exp \left\{ -\alpha \frac{(c_1 - a)^2 + (c_2 - a)^2}{b} \right\} \text{ where } a = \frac{\sqrt{R} + 1}{2} \quad b = \frac{(\sqrt{R} - 1)^2}{2} \quad (2)$$

where c_1 and c_2 are the spatial coordinates associated with the corresponding point (in the case of regular lattices $c_1, c_2 = 1, \dots, \sqrt{R}$). The parameter α controls for the degree of instability, and oscillates between 0 and 1. As a counterexample, we also introduce the case of instability without any spatial structure, so that the values of h_r will be obtained from a uniform distribution $U(-1;1)$.

The values of the variables x and ε come from two independent unit normal distributions; we assign a value of 1 both to β_1 and to β_2 . In these conditions, and using a (20×20) regular grid together with a row-standardized weighting matrix based on rook-type movements, we obtained 1000 independent draws. Every draw follows a SLM scheme of dependencies with a basic level of autocorrelation equal to 0.5 (this is the value of ρ in expression 1), but with heterogeneity according to the elliptical paraboloid of (2) or the uniform random distribution of the second case.

Next, we estimate model (1) using the *Zoom* algorithm described in López et al. (2009). This method consists in obtaining the maximum likelihood estimation (ML from now on) of the model for each point in the sample, using only the information of the, say, $(m - 1)$ nearest points to the point in question:

$$y_r^{(m)} = \rho_r^{(m)} \mathbf{W}_r^{(m)} y_r^{(m)} + x_r^{(m)} \beta_r^{(m)} + \varepsilon_r^{(m)}; \quad \varepsilon_r^{(m)} \sim N(0; \sigma_{r,m}^2 \mathbf{I}_m) \quad (3)$$

The indices r and m mean that the data correspond to the local system defined around point r ; $y_r^{(m)} = (y_r, y_{i_1}, y_{i_2}, \dots, y_{i_{m-1}})$ where $i_k \in N(r)$ and $N(r)$ is the set of indices of the $(m - 1)$ closest neighbours to point r . The same criterion is used to construct $x_r^{(m)}$. Matrix $\mathbf{W}_r^{(m)}$ is the weighting matrix obtained for this local system using the same connectivity criteria as in the case of \mathbf{W} . Finally, $\rho_r^{(m)}$, $\beta_r^{(m)}$ and $\sigma_{r,m}^2$ are the local parameters of interest. We refer to m as the *Zoom size* (equivalent to the *window size* in the literature of nonparametric methods or the *bandwidth* in the GWR literature).

Figure 1 displays the results corresponding to the average value estimated for the parameter of local spatial autocorrelation, $\hat{\rho}_r^{(m)}$, in each of the two cases. Although this is only an example, the results are interesting because they show that the local estimation has good capacity to identify the type of instability that is acting in the sample (if there is any, obviously).

The fourth point refers to the zoom size or bandwidth. In the previous example, we adopted a simple decision so that, to resolve the ML estimation of the local SLM at each point of the sample, we specified a diagonal $(R \times R)$ matrix to select the corresponding observations:

$$D^{(r)} = \left[\begin{array}{cccc} d_{1r} & 0 & \dots & 0 \\ 0 & d_{2r} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & d_{Rr} \end{array} \right] \left. \begin{array}{l} \rightarrow y_r^{(m)} = D^{(r)} y \\ \rightarrow X_r^{(m)} = D^{(r)} X \end{array} \right\} \quad (4)$$

$$d_{jr} = \begin{cases} 1 & j \in N(r) \\ 0 & j \notin N(r) \end{cases}$$

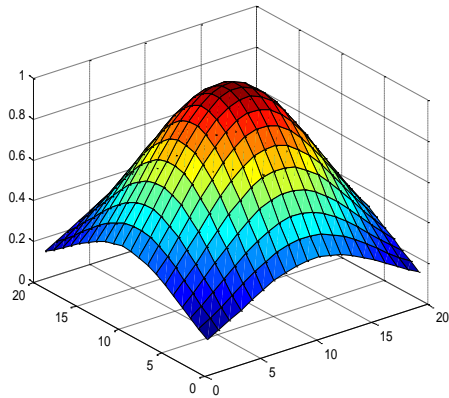
being $N(r)$ the set of nearest neighbours to point r .

In our experience, the criterion of (4) works reasonably well with small values of m (see Davidson, 2000, for a more general discussion of this kind of kernel functions). The GWR literature uses the criterion known as *cross-validation* in order to determine the most adequate specification of the

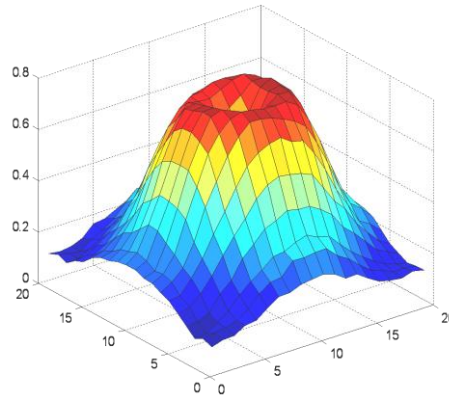
bandwidth (Fotheringham et al., 1998). This criterion consists in selecting the configuration of the bandwidth that minimises the mean squared error of prediction of the GWR estimation. The adoption of this criterion is not obvious in a pattern of simultaneous dependencies, like ours, in which case each observation influences (and receives influences) from all of its neighbours. In any case, the problem of how to determine the bandwidth in non linear models is still not solved and needs further reflection.

Figure 1: Local estimation of ρ . Lattice: (20 x 20) and Zoom = 16

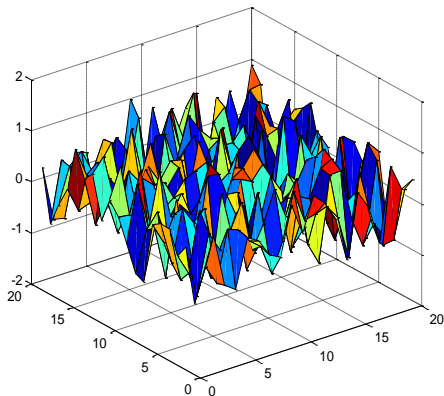
Real values corresponding to the 'h' function. The paraboloid case



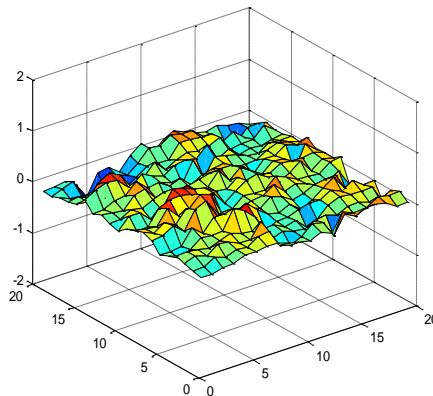
Average local estimation of ρ after 1000 draws. The paraboloid case



Real values corresponding to the 'h' function. The uniform distribution case, U(-1;1)



Average local estimation of ρ after 1000 draws. The uniform distribution case, U(-1;1)



3. LOCAL ESTIMATION OF SPATIAL INSTABILITIES

This section focuses on the modelling of the heterogeneity. The usual algorithms of local estimation, like the SALE (Pace and Lesage, 2004) or the Zoom (Mur et al., 2008), are useful in order to unveil the heterogeneity that exists in the mechanisms of spatial dependence. On occasions, information may also be available about how the break processes are acting. If this information exists, it should be used to improve the estimation of the model.

The case that we describe is when instability is related to some indicator that affects the interaction of each point with its surroundings. This is the problem analysed by Mur et al. (2008 and 2009a) and Angulo et al. (2008), where a series of tests of heterogeneity are developed, using an instability indicator. To implement the tests, it is not necessary to know the functional form that relates the parameter (unstable) to the variable, or variables, of instability. However, this information is fundamental for carrying out the estimation. Below, we propose a partial solution, which is relatively simple and which requires little information; it consists, basically, of adapting the expansion of parameters method of Casetti and Poon (1996).

The problem we wish to deal with is the lack of information about the form adopted by the function $h[-]$ in:

$$\left. \begin{array}{l} y = \rho \mathbf{H} \mathbf{W} y + X \beta + \varepsilon \\ \varepsilon \sim N(0, \sigma^2 I) \end{array} \right\} \quad (5)$$

$$\mathbf{H} = \text{diag} \{h(z_r \alpha); r = 1, 2, \dots, R\}; h(0) = \kappa < \infty$$

This function is unknown but it can be approximated by a McLaurin expansion of a high enough order:

$$h(z_r \alpha) = h(0) + h^{(1)}(0)[z_r \alpha] + \frac{h^{(2)}(0)}{2!}[z_r \alpha]^2 + \dots + \frac{h^{(n)}(0)}{n!}[z_r \alpha]^n \quad (6)$$

where $h^{(d)}$ refers to the d^{th} derivative of function h . We suppose that the arguments of function $h[-]$ are identified.

For example, if we associate the break with the Cartesian coordinates of the corresponding points, a linear approximation leads to:

$$h(z_r \alpha) \cong \gamma_0 + \gamma_1 c_1 + \gamma_2 c_2 \quad (7)$$

$$z_r = [c_1 \quad c_2]; \gamma_0 = h(0); \gamma_j = h^{(j)}(0) \alpha_j \quad j = 1, 2$$

In the case of a quadratic expansion:

$$h(z_r \alpha) \cong \gamma_0 + \gamma_1 c_1 + \gamma_2 c_2 + \gamma_3 c_1^2 + \gamma_4 c_2^2 + \gamma_5 c_1 c_2 \quad (8)$$

where the parameters are defined accordingly. Then, it suffices to substitute the approximation of (6) into (5) to ‘linealize’ the structure of matrix \mathbf{H} :

$$\mathbf{H} \cong \sum_{i=0}^n \gamma_i \mathbf{H}_{q_i} \tag{9}$$

where $\mathbf{H}_{q_i} = \text{diag}(q_{ri}, r = 1, \dots, R,)$ and q_i the corresponding variable from the change. After that, we can ‘expand’ the main equation of the SLM:

$$y = \sum_{i=0}^n \gamma_i \mathbf{H}_{q_i} \mathbf{W}y + X\boldsymbol{\beta} + \eta = \sum_{i=0}^n \gamma_i \mathbf{W}_{q_i} y + X\boldsymbol{\beta} + \eta \tag{10}$$

where $\mathbf{W}_{q_i} = \mathbf{H}_{q_i} \mathbf{W}$. The error term η is the sum of the original error, ε , and of the approximation errors committed in relation to matrix \mathbf{H} . In general terms, model (10) coincides with the model proposed by Huang (1984) although, in this case, using a very particular sequence of linearly independent weighting matrices.

In our case, using the parameterization of the \mathbf{W} matrix given in (9), it is relatively simple to obtain a Lagrange Multiplier statistic that tests the linear or non-linear restrictions on the parameters of the model (details of matrix information and score in Appendix II.a). For example, the null hypothesis that all the parameters associated with the expansion are zero:

$$\left. \begin{array}{l} H_0 : \gamma_1 = \dots = \gamma_n = 0 \\ H_A : \text{no } H_0 \end{array} \right\} \tag{11}$$

corresponds to the case where there is no heterogeneity in the behaviour of the parameter of spatial dependence, ρ . The resulting statistic is the so-called $\text{LM}_{\text{Break}}^{\text{SLM}}$, in its raw version (Mur et al., 2008), or $\text{LM}_{\text{Break}}^{*\text{SLM}}$ in its robust version (Angulo et al., 2008). We should equally point out that the rejection of the null hypothesis of (11) may be due to other factors such as, for example, a non adequate selection of the basic determining elements (variables z) or to a poor approximation to function $h(-)$.

We use the Lagrange Multiplier associated to the null hypothesis of (11) to evaluate the quality of the approximation to the unknown function $h[-]$. In other words, returning to the example of expressions (7) and (8), the question is whether a simple linear approximation is sufficient to explain the symptoms of instability, detected using the Cartesian coordinates, as in (7), or whether it is necessary to adopt more complex expansions, like the quadratic one of (8). The rejection of (11) leads us, in the first place, to propose a simple linear approximation, like that of (7). Model (8) should be the following step. Both specifications are related by the null hypothesis:

$$\left. \begin{array}{l} H_0 : \gamma_3 = \gamma_4 = \gamma_5 = 0 \\ H_A : \text{no } H_0 \end{array} \right\} \quad (12)$$

The test statistic will be obtained, as usual, as the quadratic form of the score vector over the inverse of the information matrix of (detail are provided in Appendix II.b):

$$LM_{(3,4,5)}^{break} = [g(y; \theta)_0]' [I(\theta)_0]^{-1} [g(y; \theta)_0]_{as} \tilde{\chi}^2(3) \quad (13)$$

4. MONTE-CARLO EVIDENCE

In the previous section, we have expressed our interest in the problem of choosing an adequate functional form to capture the pattern of instability that affects the autocorrelation coefficient.

We have obtained several instability tests, as a necessary first step in the process of modelling the instability of the parameter ρ . To proceed in this direction, we need, at least, information about the variables that are acting on the break (we refer to them as break indicators). In what follows, we assume that the parameter of dependencies evolves over space according to the geographical coordinates of each point, and our intention is to model the instability.

The procedure we suggest consists in three stages:

(i) ‘Linealize’ the break, as in (7) and (8), in order to solve the corresponding tests. The coordinates of the centroid of each cell will be used as break indicators. We employ the LM_{Break}^{SLM} test to check for the two restrictions of (14):

$$H_0 : \gamma_1 = \gamma_2 = 0 \quad (14)$$

(ii) Rejection of the above null hypothesis should be treated as evidence in favour of a break which admits, at least, a linear approximation. The problem now is to discuss whether the linear approximation of (7) is enough to tackle the problem. A second order polynomial in the geographical coordinates, as in (8), is a more general specification which leads us to a new instability test. The restrictions in this case will extend to five parameters:

$$H_0 : \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 0.$$

(iii) Statistic (13) allows us to complete the discussion about the adequacy of the linear approximation. The null hypothesis ($H_0 : \gamma_3 = \gamma_4 = \gamma_5 = 0$) implies that a first order polynomial (linearity) is enough whereas the alternative hypothesis requires a second order polynomial.

Table 1 presents the main results obtained from the Monte-Carlo experiment. The first column indicates the type of break introduced into the parameter of spatial dependence. Particularly, we have tried out very simple mechanisms of break of a discrete type, in a North-South (H2) or Centre-Periphery (H3) regime, first order (H4) and second order (H5) order polynomial processes, wavy (as in H6 to H8) and also random processes of instability (H1) where the parameter of local dependencies comes from a uniform distribution without a spatial pattern. The details of these functions appear in the Appendix I.

H0 is the Control Case (the parameter is constant over space). The conclusion here is that there do not appear to be problems with the size of the tests. The sequence of Multipliers works well, especially for the case proposed in Section 3. According to these results, we can be confident in distinguishing between breaks that follow a first or a second order polynomial in the spatial coordinates (models H4 and H5).

**Table 1: Size and power of the instability tests.
A selection of cases of interest.**

	Regular Lattice 7X7			Regular Lattice 20x20		
	LM ^{SLM} _{Break} ⁽¹⁾	LM ^{SLM} _{Break} ⁽²⁾	LM ^{Break} _(3,4,5)	LM ^{SLM} _{Break} ⁽¹⁾	LM ^{SLM} _{Break} ⁽²⁾	LM ^{Break} _(3,4,5)
H0	0.039	0.053	0.063	0.042	0.046	0.046
H1	0.118	0.179	0.153	0.192	0.268	0.190
H2	0.910	0.484	0.503	1.000	1.000	1.000
H3	0.000	1.000	0.979	0.018	1.000	1.000
H4	1.000	1.000	0.055	1.000	1.000	0.042
H5	0.000	1.000	0.878	0.000	1.000	0.945
H6	0.480	0.469	0.384	1.000	1.000	0.998
H7	0.267	0.424	0.349	0.002	0.003	0.023
H8	0.035	0.010	0.002	0.023	0.030	0.022

(1) Break indicators: LINEAR TREND OF THE COORDINATES.

(2) Break indicators: SECOND ORDER POLINOMIAL OF THE COORDINATES.

The problems appear in other aspects such as, for example, the identification of random mechanisms (H1). The tests, in this case, perceive some traces of instability but the signs are too weak to support a strategy to identify the nature of the break. This result is no surprise given that the break indicators introduced into the tests (geographical coordinates) have no relation with the nature of the break that really exists in the model (totally random). In the cases of a discrete break (H2 and H3), the situation is a bit confusing. Linear patterns are well adapted to North-South regimes whereas Centre-Periphery regimes appear to require polynomials of a higher order. Of course, there are several other factors which have an impact on these results (how the clubs are defined, where they are located, etc) and that must be taken into account. Finally, cases H6, H7 and H8 are used as counter-examples in the sense that they are very far

from the ideal conditions explored in Section 3. It is clear that, as we introduce stronger nonlinearities into the break patterns, the performance of the battery of tests is greatly reduced. In fact, sample size has hardly any effect on the behaviour of the tests.

5. SOME APPLICATIONS

The literature on spatial models has dealt, on several occasions, with problems of instability similar to ours. In most cases, the solution has been the introduction of a discrete break, depending on the geographical location of each point. The final result is a kind of club structure that seems too rigid. Below, we look again at two applications in which the topic of instability plays a crucial role.

The first comes from Anselin (1988, chapter 12) and corresponds to the example of the determinants of neighbourhood crime in Columbus, Ohio. The second example consists in the work of Mur et al. (2008), who identify a Centre-Periphery break in the mechanisms of spatial dependence of the per capita income in Europe.

5.1. The classical example of crime (Anselin 1988)

The author relates the crime variable in 1980 in Columbus (CRIME defined as residential burglaries and vehicle thefts per thousand households in the 49 neighbourhoods of the sample) with INCOME and HOUSING values in thousands of dollars. The basic model offers clear signs of misspecification due to an omitted spatial lag but, '*when spatial dependence is acknowledged, evidence is found for structural instability*' (p. 200). There are symptoms of heteroskedasticity and also of an East-West trend in the spatial expansion of the parameters.

The SLM estimated by Anselin appears in the first column of Table 2, under the heading of equation (2.1). Equation (2.2) corresponds to the ML estimation of this model but with a linear spatial instability pattern in the parameter associated with the spatial lag. In the last column, we apply a quadratic expansion to this coefficient. According to the Lagrange Multipliers, the evidence of spatial instability is weak although it points towards a nonlinear scheme. The battery of likelihood ratios (LR in what follows) at the bottom of the table, confirms this impression. All the coefficients in equation (2.3) are highly significant and have the right sign.

The map of spatially varying estimated lag coefficients appears in the left-hand panel of Figure 2. There is a strong clustering of high values in the central neighbourhoods of Columbus, with values between 0.25 and 0.30. The intensity of this spatial interaction decreases as we move towards the periphery of the city, obtaining negative estimates in the neighbourhoods situated in the most external rings. Not surprisingly, the LISA measures that appear in the right-hand panel show a similar picture (high-high connections in the centre of the city and non-significant or even low/high and high/low relations towards the periphery).

Table 2: Determinants of neighbourhood crime in Columbus (Anselin 1988)

	Equation (2.1)	Equation (2.2)	Equation (2.3)
CONSTANT	45.0571 (6.28)	45.4362 (6.90)	53.2119 (9.23)
INC	-1.0307 (-3.38)	-1.1918 (-3.98)	-0.8567 (-2.78)
HOUSE	-0.2660 (-3.01)	-0.2308 (-2.61)	-0.1745 (-2.22)
W_CRIME	0.4314 (3.67)	0.4538 (4.02)	0.2849 (2.95)
γ_1		0.0127 (1.55)	0.009 (0.1036)
γ_2		-0.0074 (-0.87)	0.0051 (0.57)
γ_3			-0.0048 (-3.76)
γ_4			-0.0068 (-4.35)
LM _{Break} ^{SLM (1)}	2.38 (<i>p</i> -val: 0.3042)		
LM _{Break} ^{SLM (2)}	6.52 (<i>p</i> -val: 0.1635)		
LM _(3,4,5) ^{Break}		6.16 (<i>p</i> -val: 0.0450)	
Log-likelihood	-182.39	-181.00	-173.97
LR (eq 2.1 vs 2.2)	2.80 (<i>p</i> -val: 0.2464)		
LR (eq 2.1 vs 2.3)			16.86 (<i>p</i> -val: 0.0021)
LR (eq 2.2 vs 2.3)		14.06 (<i>p</i> -val: 0.0009)	

(1) Break indicators: LINEAR TREND OF THE COORDINATES.

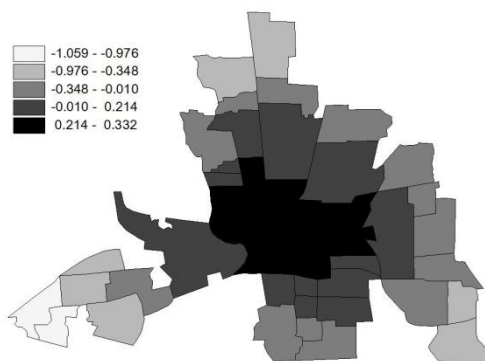
(2) Break indicators: SECOND ORDER POLINOMIAL OF THE COORDINATES.

5.2. The case of Mur et al. (2008)

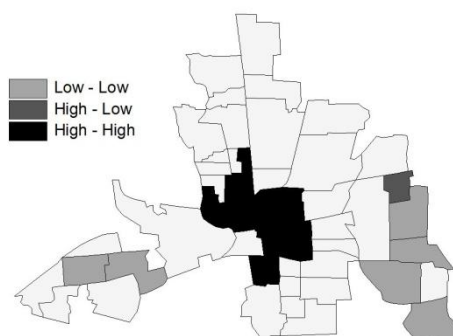
This case comes from Mur et al. (2008) where the authors study the spatial distribution of per capita income in Europe in the year 2004 (variable INCOME), using NUTS III regions. The authors introduce the population density, (DENSITY), and the weight of the agricultural sector in the regional product, (AGRI_WEIGHT) in the right-hand side of the equation. The estimation of the simple linear model offers clear signs of misspecification that lead to a SLM, whose estimation appears in the first column of Table 3, under the heading of equation (3.1). In equation (3.2), we estimate a model with a linear pattern of instability in the parameter of spatial dependence, which equation (3.3) generalizes into a second order polynomial pattern.

Figure 2: Determinants of neighborhood crime in Columbus (Anselin 1988)

Map of the estimated coefficients (eq. 2.3)



LISA Measures



The authors find evidence of instability in model (3.1) that they interpret as a Centre-Periphery discrete break as it is shown in Figure 3.a, because the interaction seems to be stronger in external zones of the continent. Moreover, the results of Table 3 point towards a nonlinear break in this model. The ample model of (3.3) is clearly superior to the other two, whatever the criteria applied.

Figure 3.b shows the map of the local estimates using a linear expansion in the coefficient of the spatially lagged income. The map associated with equation 3.3 appears in Figure 3.c. A large number of regions, located at the centre of the continent (depicted in white, 754 regions out of a total of 1274), have an intermediate value in this coefficient. The intensity of the dependence decreases as we move towards the East or the West of the continent but

increases if we move North or South. The higher levels correspond to various Mediterranean points together with the northernmost Swedish regions.

Table 3: Income per capita in the European regions (Mur et al., 2008)

	Equation (3.1)	Equation (3.2)	Equation (3.3)
CONSTANT	3.4872 (17.47)	3.6143 (17.50)	4.2404 (18.22)
AGRI_WEIGHT	-0.0236 (-13.95)	-0.0256 (-13.14)	-0.0250 (-11.54)
DENSITY	0.0947 (13.19)	0.0949 (11.62)	0.1004 (11.97)
W_INCOME	0.6488 (32.49)	0.6727 (32.55)	0.9927 (20.67)
γ_1	--	-0.0001 (-1.33)	0.0006 (2.99)
γ_2	--	-0.0007 (-4.59)	-0.0166 (-7.78)
γ_3	--	--	-0.0001 (-4.95)
γ_4	--	--	0.0002 (7.50)
$LM_{Break}^{SLM(1)}$	25.67 (<i>p</i> -val: 0.0000)		
$LM_{Break}^{SLM(2)}$	70.39 (<i>p</i> -val: 0.0000)		
$LM_{(3,4,5)}^{Break}$		49.48 (<i>p</i> -val: 0.0000)	
Log-likelihood	-110.49	-97.77	-69.73
LR (eq 3.1 vs 3.2)	25.45 (<i>p</i> -val: 0.0000)		
LR (eq 3.1 vs 3.3)		81.53 (<i>p</i> -val: 0.0000)	
LR (eq 3.2 vs 3.3)			56.08 (<i>p</i> -val: 0.0000)

(1)Break indicators: LINEAR TREND OF THE COORDINATES.

(2)Break indicators: SECOND ORDER POLINOMIAL OF THE COORDINATES.

6. CONCLUSION AND FUTURE PROSPECTS

Space tends to be diverse and singular, a characteristic that facilitates the existence of peculiarities in econometric models. Instability is one of these characteristics. The severe consequences of not taking into account the lack of stability in spatial relationships require the development of a research line into this topic. The literature in spatial econometrics has dealt with this question on different occasions, although the discussion has mainly been limited to the regression coefficients. However, the mechanisms of spatial dependence also present clear signs of instability, as reflected by the LISA measures.

Figure n°3: The Mur et al (2008) study

Figure 3.a: Discrete break. Map of the estimated coefficients

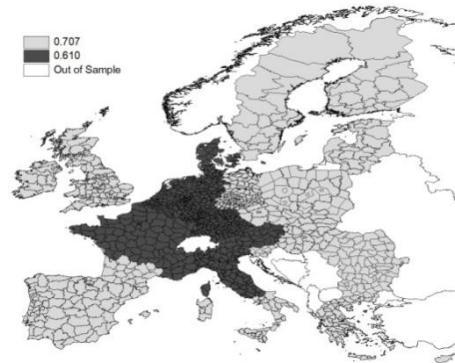


Figure 3.b: Map of the estimated coefficients (eq. 3.2)

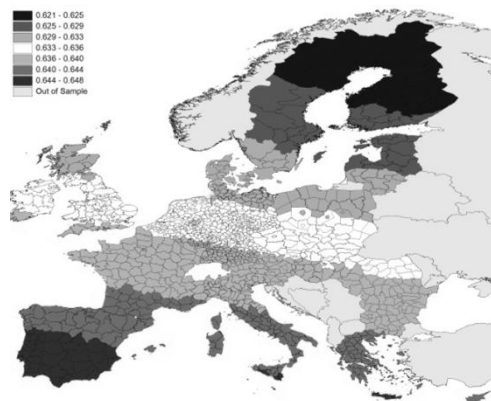
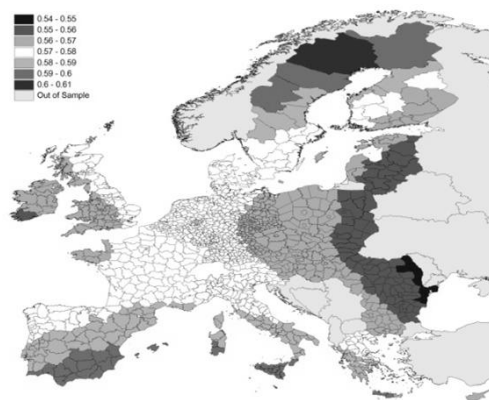


Figure 3.c: Map of the estimated coefficients (eq. 3.3)



Local estimation algorithms are an interesting technique to measure the impact of the heterogeneity as shown, for example, by López et al. (2009). The purpose of our paper is to offer support for the usefulness of this technique and to develop some new tools. The solution has a computational cost but appears to be effective. The question of what to do with the results of the local estimation, once we confirm the existence of instability in the mechanisms of spatial dependence, is really complex. In our opinion, the objective must be to model the instability in order to solve the specification. This is the purpose of the techniques analysed in the paper, which allow us to begin the discussion. However, the solution is not completely satisfactory. Clearly, more work is needed in order to obtain an overall and compact treatment of the topics of nonlinearity and instability in space and in spatial econometric relationships.

APPENDIX I

Functional forms used in the experiment

We have used several functional forms in the Monte Carlo exercise in order to develop the idea of continuous instability in the parameter of spatial dependence. The principal element of this discussion is the diagonal \mathbf{H} matrix, expression (5), which captures the different assumptions of instability introduced into the experiment.

To simplify the discussion, we have assumed that, in all cases, the instability has a geographical basis. This assumption means that the parameter of dependence evolves regularly over space depending on the coordinates of the corresponding point. In what follows, we will refer to c_1 as the abscissa of the centroid of each cell of the lattice and c_2 as the ordinate of this point on a hypothetical coordinate axis. Moreover, as a control case, we will use a SLM whose parameter of dependence remains stable, and equal to 0.5. In the other cases, there is an ‘average level’ of dependence for the lattice close to the mark of 0.5. The functional forms used in the simulation appear in Table A.1, where R is the sample size, $a = \frac{\sqrt{R} + 1}{2}$ and c_1 and c_2 are the coordinates of the centroid of each cell ($c_1, c_2 = 1, \dots, \sqrt{R}$).

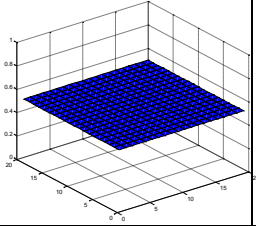
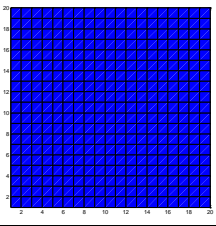
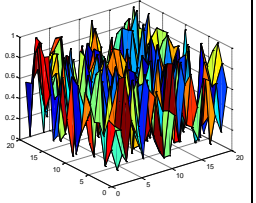
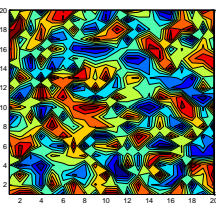
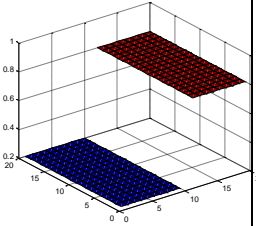
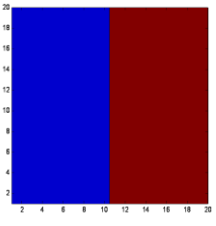
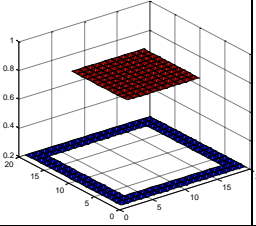
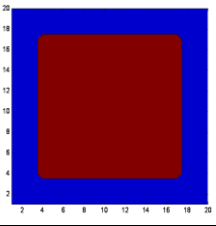
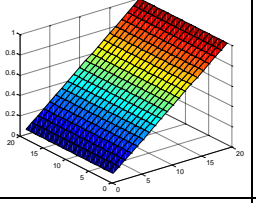
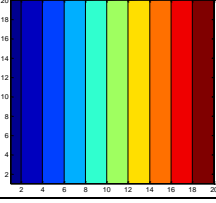
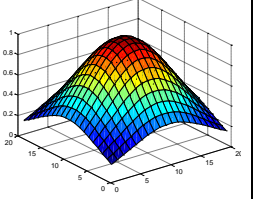
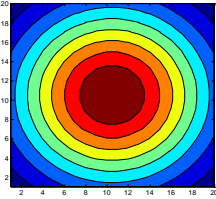
APPENDIX II.a:

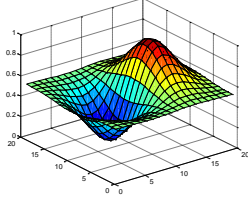
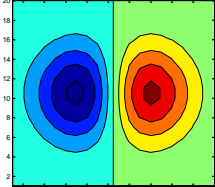
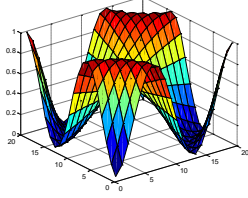
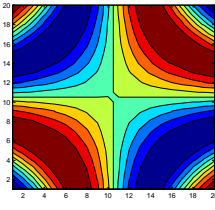
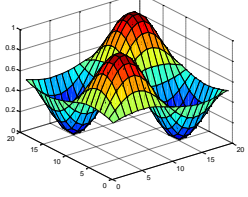
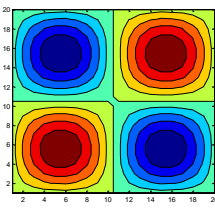
Matrix information and score vector

After solving the approximation of (9), it is possible to obtain the ML estimation of the resulting model. The log-likelihood function is standard:

$$l(y; \theta) = -\frac{R}{2} \ln(2\pi) - \frac{R}{2} \ln(\sigma^2) + \ln|A| - \frac{1}{2\sigma^2} (\mathbf{A}\mathbf{y} - \mathbf{X}\boldsymbol{\beta})' (\mathbf{A}\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \quad (\text{A.1})$$

Table A.1: Instability functions

	Function	3D perspective	2D perspective
<i>H0</i> Control Case	$h(\rho, c_1, c_2) = 0.5$		
<i>H1</i> Random instability	$h(\rho, c_1, c_2) = u$ where $u \in U(0,1)$		
<i>H2</i> Discrete North-South	$h(\rho, c_1, c_2) = \begin{cases} 0.2 & \text{if } 1 \leq c_2 \leq \frac{R}{2} \\ 0.8 & \text{if } \frac{R}{2} < c_2 \leq R \end{cases}$		
<i>H3</i> Discrete Centre- Periphery	$h(\rho, c_1, c_2) = \begin{cases} 0.8 & \text{if } (c_i - a) \leq c \\ & (i = 1, 2) \\ 0.2 & \text{in other case} \end{cases}$		
<i>H4</i> Plane with slope	$h(\rho, c_1, c_2) = \rho \frac{c_2}{\sqrt{R}}$		
<i>H5</i> Paraboloid	$h(\rho, c_1, c_2) = e^{-\rho((c_1 - a)^2 + (c_2 - a)^2)}$		

	Function	3D perspective	2D perspective
<i>H6 Opposite Paraboloids</i>	$h(\rho, c_1, c_2) = \rho(c_2 - a)e^{-\rho(c_1 - a)^2 + (c_2 - a)^2} + \frac{1}{2}$		
<i>H7 Centered Waves</i>	$h(\rho, c_1, c_2) = \frac{\sin(\rho(c_1 - a)^2(c_2 - a)^2) + 1}{2}$		
<i>H8 Dispersed Waves</i>	$h(\rho, c_1, c_2) = \frac{\sin(\rho(c_1 - a)^2)\sin(\rho(c_2 - a)^2) + 1}{2}$		

where θ is the vector of parameters of the model $\theta = [\beta, \gamma_i, \sigma^2]'$ and \mathbf{A} is a matrix $\mathbf{A} = \mathbf{I}_R - \sum_{i=0}^k \gamma_i \mathbf{W}_{q_i}$. The score vector can be written as:

$$g(y; \theta) = \frac{\partial l(y; \theta)}{\partial \theta} = \frac{1}{\sigma^2} \begin{bmatrix} (\mathbf{A}\mathbf{y} - \mathbf{X}\boldsymbol{\beta})' \mathbf{X} \\ \eta' \mathbf{W}_{q_i} \mathbf{y} - \sigma^2 \text{tr} \mathbf{A}^{-1} \mathbf{W}_{q_i} \\ (i=0, 1, \dots, n) \\ -\frac{R}{2} + \frac{\eta' \eta}{2\sigma^2} \end{bmatrix} \quad (\text{A.2})$$

where $\eta = \mathbf{A}\mathbf{y} - \mathbf{X}\boldsymbol{\beta}$. The Hessian matrix can easily be obtained:

$$\begin{aligned}
 & \frac{\partial^2 l(y^R; \theta)}{\partial \theta \theta'} = \\
 & -\frac{1}{\sigma^2} \begin{bmatrix}
 \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W}'_{q_i} y & \frac{1}{\sigma^2} \mathbf{X}'\eta \\
 \begin{matrix} y'\mathbf{W}'_{q_i} \mathbf{X} \\ (i=0,1,\dots,n) \end{matrix} & \begin{matrix} y'\mathbf{W}'_{q_i} \mathbf{W}_{q_j} y + \sigma^2 \text{tr}(\mathbf{A}^{-1} \mathbf{W}_{q_i} \mathbf{A}^{-1} \mathbf{W}_{q_j}) \\ (i,j=0,1,\dots,n) \end{matrix} & \begin{matrix} \frac{1}{\sigma^2} y'\mathbf{W}'_{q_i} \eta \\ (i=0,1,\dots,k) \end{matrix} \\
 \frac{1}{\sigma^2} \eta' \mathbf{X} & \frac{1}{\sigma^2} \eta' \mathbf{W}_{q_i} y & -\frac{R}{2\sigma^2} + \frac{\eta' \eta}{\sigma^4}
 \end{bmatrix} \quad (\text{A.3})
 \end{aligned}$$

The information matrix corresponds to the negative expectation of the previous expression:

$$\begin{aligned}
 & I(\theta) = -E \left[\frac{\partial^2 l(y; \theta)}{\partial \theta \theta'} \right] = \\
 & = \frac{1}{\sigma^2} \begin{bmatrix}
 \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W}'_{q_i} \mathbf{A}^{-1} \mathbf{X} \beta & 0 \\
 \begin{matrix} \beta' \mathbf{X}' \mathbf{A}^{-1} \mathbf{W}'_{q_i} \mathbf{X} \\ (i=0,1,\dots,n) \end{matrix} & \begin{matrix} \beta' \mathbf{X}' \mathbf{A}^{-1} \mathbf{W}'_{q_i} \mathbf{W}_{q_j} \mathbf{A}^{-1} \mathbf{X} \beta + \\ + \sigma^2 \text{tr} \mathbf{A}^{-1} \mathbf{W}_{q_i} \mathbf{A}^{-1} \mathbf{W}_{q_j} + \sigma^2 \text{tr} \mathbf{A}^{-1} \mathbf{W}'_{q_i} \mathbf{A}^{-1} \mathbf{W}_{q_j} \\ (i,j=0,1,\dots,n) \end{matrix} & \text{tr} \mathbf{W}_{q_i} \mathbf{A}^{-1} \\
 0 & \text{tr} \mathbf{W}_{q_i} \mathbf{A}^{-1} & \frac{R}{2\sigma^2}
 \end{bmatrix} \quad (\text{A.4})
 \end{aligned}$$

APPENDIX II.b: Matrix Information and score vector

The null hypothesis is:

$$\left. \begin{aligned}
 & H_0 : \gamma_3 = \gamma_4 = \gamma_5 = 0 \\
 & H_A : \text{no } H_0
 \end{aligned} \right\} \quad (\text{A.5})$$

We need to obtain the corresponding Multiplier, for which it will be necessary to evaluate the score vector in the H_0 of (A.5):

$$g(y; \theta)|_{H_0} = \frac{\partial l(y; \theta)|_{H_0}}{\partial \theta} = \frac{1}{\sigma^2} \begin{bmatrix} \mathbf{0}_{(k \times 1)} \\ \mathbf{0}_{(3 \times 1)} \\ \eta_0' \mathbf{W}_{q_i} y - \sigma^2 \text{tr} \mathbf{A}_0^{-1} \mathbf{W}_{q_i} \\ (i=3,4,5) \\ 0 \end{bmatrix} \quad (\text{A.6})$$

where $\mathbf{A}_0 = \mathbf{I}_R - \gamma_0 \mathbf{W} - \gamma_1 \mathbf{W}_{q_1} - \gamma_2 \mathbf{W}_{q_2}$ and $\eta_0 = \mathbf{A}_0 y - \mathbf{X} \beta$. The information matrix must also be evaluated in the null hypothesis:

$$I(\theta)_0 = \frac{1}{\sigma^2} \begin{bmatrix} \mathbf{X}' \mathbf{X} & \mathbf{X}' \mathbf{W}'_{q_i} \mathbf{A}_0^{-1} \mathbf{X} \beta & 0 \\ (i=0,1,\dots,5) & \beta' \mathbf{X}' \mathbf{A}_0^{-1} \mathbf{W}'_{q_i} \mathbf{W}_{q_j} \mathbf{A}_0^{-1} \mathbf{X} \beta + & \\ & + \sigma^2 \text{tr} \mathbf{A}_0^{-1} \mathbf{W}_{q_i} \mathbf{A}_0^{-1} \mathbf{W}_{q_j} + \sigma^2 \text{tr} \mathbf{A}_0^{-1} \mathbf{W}'_{q_i} \mathbf{A}_0^{-1} \mathbf{W}_{q_j} & \text{tr} \mathbf{W}_{q_i} \mathbf{A}_0^{-1} \\ (i,j=0,1,\dots,5) & & (i=0,\dots,5) \\ 0 & \text{tr} \mathbf{W}_{q_i} \mathbf{A}_0^{-1} & \frac{R}{2\sigma^2} \\ (i=0,1,\dots,5) & & \end{bmatrix} \quad (\text{A.7})$$

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CARTES DE DÉPENDANCE SPATIALE CONTINUE

Résumé - L'hétérogénéité est un problème fréquent dans les modèles économétriques spatiaux avec de sérieuses conséquences pour l'inférence statistique. Cet article s'intéresse aux problèmes induits par l'existence d'instabilités dans le mécanisme de la dépendance spatiale dans un modèle spatial autorégressif, en supposant que tous les autres termes de la spécification restent stables. Nous commençons par discuter le rôle joué par les algorithmes d'estimation locale pour détecter les instabilités. Ensuite, il est problématique de décider ce qu'il convient de faire une fois l'hétérogénéité détectée. La réaction logique consiste à paramétrer cette absence de stabilité. Cependant, la solution n'est pas immédiate. Dans ces conditions, en supposant qu'un ensemble d'indicateurs reliés au problème ont été identifiés, nous proposons une technique simple permettant de traiter cette forme fonctionnelle inconnue. Finalement, nous présentons quelques résultats de simulations de Monte-Carlo et une application permettant d'évaluer l'instabilité des mécanismes de dépendance spatiale au sein du processus de convergence des régions européennes.