Abstract - This paper presents a macroeconomic approach to sustainable growth. After clarifying the concept of sustainability, the interdependence between natural resources and accumulated capital stocks such as physical, human, and knowledge capital is discussed. The conditions for the substitution process leading to sustainable development are demonstrated in a one-sector approach and two versions of a multi-sector endogenous growth model. It turns out that prices of natural inputs have their major impact on growth by changing an economy's sectoral structure. The prediction of a successful substitution of knowledge for natural resources emerges to be realistic, provided that the sectoral adjustment cost in the economy are not too high.

Keywords - SUSTAINABILITY, ENDOGENOUS GROWTH, NATURAL RESOURCES

JEL Classification: Q20, Q32, O31, O41

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1. INTRODUCTION

Ten years after the well-known "Brundlandt report" was published (World Commission 1987), the notion of sustainability is widely accepted but still lacking in concise meaning. The variety of using sustainability as a desirable characterisation of long-term development is documented in different surveys (see e.g. Pezzey, 1989; Repetto, 1992; Toman, 1994; Toman/ Pezzey/Krautkraemer, 1995). According to some literature, sustainability calls for the natural environment to be saved for future generations. In their well-known contribution, Pearce/Barbier/Markandya (1990, p. 23) interpret the term as "requiring some constancy in the stock of natural environmental assets". In contrast, it is argued that sustainable paths are, in principal, not different from optimal paths in the tradition of optimal control theory (see e.g. Dasgupta, 1995; Nordhaus, 1992). A third approach is to follow the anthropocentric view of optimal control, but to add the requirement of intergenerational fairness. As a consequence, non-declining utility appears as a constraint for development paths to be sustainable. This emphasis on the undiscounted relative welfare of generations is favoured most prominently by Pezzey (1989) and will be used in this paper.

In the interdisciplinary discussion among economists, ecologists, and ethicists on sustainability, recent methods and results of macrodynamic analysis have not been fully integrated yet. It seems to be quite obvious that the improved understanding of growth processes due to so-called new growth theory is of great importance for the issue under consideration; therefore, it should be fully incorporated in environmental research. This paper aims to do so. It analyses sustainability from a macroeconomic perspective, i.e. it develops a concept of sustainability in macroeconomic terms and shows, in a coherent framework, under what conditions a sustainable development is achieved. The two basic themes which are the current generation's responsibility to future generations and the degree of substitutability between natural capital and accumulated capital like machines, human capital, and knowledge capital are extensively treated. Furthermore, the paper studies whether compensatory investments for the declining natural capital stock are feasible, and under which conditions these investments are undertaken.

In literature, there exist several contributions to the issue of endogenous growth when negative externalities are present, see Gradus/Smulders (1993), Bovenberg/Smulders (1995), and Van Ewijk/Van Wijnbergen (1995). But none of these papers deals with research and development and its impact on growth and
sustainability, which is obviously an important topic. The second part of the paper considers this issue, i.e. it studies a growth process where the stock of knowledge increases and partly substitutes for natural inputs. Different aspects of this substitution process are highlighted in two model versions of expanding goods varieties. In the first version, it will turn out that the elasticities of substitution between labour and the natural inputs are decisive for long-term development. In the second version, it will be shown that a decrease in the supply of natural inputs can even increase the steady-state rate of growth. These and other conclusions are directed towards concrete policy measures that are derived from the theoretical analysis. In general, the paper uses the methodology of neo-classical economics, which has the advantages of consistent argumentation and direct policy proposals. These virtues have been widely appreciated in static environmental theory.

The paper is organised as follows. Section 2 presents a macroeconomic approach to capture the essence of the term "sustainable growth". In Section 3, the interactions between renewable and non-renewable natural resources on the one side and accumulated capital stocks such as physical, human, and knowledge capital on the other side are laid out. Section 4 deals with the substitution of natural resources in a one-sector framework. In section 5, research and development is introduced in two versions of a three-sector growth model with research expanding the variety of goods. Section 6 derives the conditions for the substitution process leading to sustainable growth for the first model version. Section 7 presents the second version of the model, which yields additional insights for the conditions of substitution in production. Section 8 gives some extensions and section 9 concludes.

2. ASSESSMENT OF DIFFERENT DEVELOPMENT PATHS

To express the two main categories of thinking in macroeconomic terms, I distinguish between "stock" and "flow" concepts of sustainability. Stock concepts concentrate on the preservation of specific capital stocks, while flow concepts are concerned with the long-term development of macroeconomic flow variables such as individual welfare. The most obvious stock concept is the notion that, in order to call a development sustainable, a constant aggregate stock of natural capital has to be saved for future generations. In a further step, certain disaggregated natural stocks to be protected can be specified, in order to minimise environmental risks or to give future generations the option to enjoy a wide range of environmental services. A different stock that can be considered is an aggregate of natural and accumulated capital such as physical, human, and knowledge capital.

However, many economists take a different view. Their approach is to focus on human welfare and production efficiency. According to Dasgupta (1995,
p. 116), any stock concept is a "category mistake" because, in his view, it mixes up the determinants of human well-being and the constituents of well-being. He states that the preservation of the environment, which society desires, ought to be derivable from an appropriate optimisation exercise. To realise this approach in optimal control theory, it is normally assumed that the utility of the following generations is included in the utility functions of current generations; the same can be done for the stock of nature. As a result of optimisation, it will be optimal to use natural resources less intensively. But it can be shown that, if some natural inputs in production are non-renewable yet essential in production, optimal solutions often result in declining individual utility in the distant future. It also may well be "optimal" from today's perspective to gradually reduce the stock of nature to a very low level, which can pose serious problems for future generations. The same holds true for steadily increasing environmental pollution in the long run.

In response to these predictions, Howarth and Norgaard (1992) argue that the problem of intergenerational equity must be viewed as an issue of ethics that is distinct from economic efficiency in the Pareto sense. However, there is no concept of distributive justice that is widely accepted. It is nevertheless appropriate to conclude from the discussion so far that, for the maximisation of the present value of per-capita utility, sustainability requires the observation of an additional ethical constraint. This constraint says that per-capita welfare should never decline between generations, i.e. the average individual in future generations should be left no worse off than the average individual in current generations (see Riley, 1980; Pezzey, 1989 and 1994; Tietenberg, 1994; and Toman/Pezzey/Krautkraemer, 1995, for similar statements). This approach is the most commonly used flow concept of sustainable growth. Thereby, per-capita utility is only one of the possible flow variables to measure sustainability, but as a measure of welfare, it is better suited than per-capita income or consumption. The distinction between the term "development", which concentrates on utility, and "growth", which concentrates on output, does not seem necessary from a macroeconomic perspective, as it is unambiguously individual utility that is the final objective in any strand of economic theory.

The presented flow concept is equivalent to a stock concept, if we assume (i) that capital is defined in a broad sense including natural and accumulated capital, (ii) that production functions exhibit sufficient and smooth substitution possibilities between natural and man-made inputs, and that (iii) sufficient knowledge about future production techniques is available today. Assumptions (i) and (ii) are explicitly used in many models such as Solow (1986), whereas (iii) has been a point of severe critique by ecologists. It is a weakness of many economic studies, such as Nordhaus (1991) on the greenhouse effect, that the uncertainties...
caused by a lower natural capital stock are not considered. In fact, it must be acknowledged that in some cases large-scale environmental assets or risks are inherently difficult to value. Aggregate losses of diminishing natural resources are high whenever there are substantial irreversibilities of environmental damages. According to Arrow/Fisher (1974), the expected benefit of an irreversible decision should be adjusted to reflect the loss of options it entails. The lower the information on possible irreversibilities, the more serious the situation becomes. In terms of macroeconomic theory, irreversibilities lead to nonsubstitutabilities in the production function. More specifically, it means that the substitution of accumulated resources for natural capital is limited in some fields. As a consequence, smooth production functions cannot be used to model some parts of the economy. For environmental resources with a high uncertainty about irreversibilities, it is appropriate to propose that the corresponding resource stock should not decline over time. Put differently, with low information and a high potential asymmetry in the aggregate loss function representing the cost of certain environmental damages, a safe minimum standard for specific environmental resources may be optimal. To analyse this issue more thoroughly in future research, irreversibilities and uncertainties must be considered more carefully, so that the consequences for human welfare can be better predicted. One of the attempts in economics is Baranzini/Bourguignon (1995).

Two points should be emphasised here. First, the safe minimum standards for specific environmental stocks are only the means to achieve sustainability in the form of a constant or rising flow of undiscounted individual welfare; they are not the final target. Second, safe minimum standards concern only parts of nature, whereas the demand for a constant aggregate natural capital stock is clearly inefficient regarding production techniques. The exclusive substitution of renewable natural resources for non-renewable natural resources is not justified, either based on an efficiency or a fairness criterion. Oil, for example, will in the future not be primarily replaced by timber or sugar-cane, but by solar and wind energy, which both require substantial inputs of physical and knowledge capital, i.e. non-natural capital. Also, regarding the imminent problems of developing countries today, it becomes clear that an increase in welfare cannot be achieved in the future by simply preserving the stock of nature. The following proposition summarises the findings so far.

**Proposition 1:** Sustainability is more than a concept concerning the preservation of nature; its primary focus is the long-term development of undiscounted individual welfare. The higher the probability of irreversibilities and the larger the uncertainties about aggregate costs of damages, the more safe minimum standards for certain natural capital stocks become a rational means to
obtain sustainability.

In principal, there exist three types of development paths. The first type of path is reached under free market conditions. In economics, the allocative efficiency of free markets has been extensively described. But it also has been emphasised that positive and negative externalities distort the efficient allocation and lower aggregate welfare. Learning effects in the sense of Marshall (1920) belong to the category of positive externalities; environmental damage without compensation from markets constitutes negative externalities. If all external effects (positive or negative) are internalised, the economy reaches the second type of path, which is the "optimal" path. "Optimal" is as viewed from the perspective of today's generations, which use a positive discount rate for future utility streams. The third type of path is the sustainable development path on which undiscounted individual utility does not decline between generations. Figure 1 shows the relation between the three types of paths in an overview. It is expressed that free market and "optimal" paths are distinguished by the existence of externalities and that both of these paths can be either sustainable or non-sustainable.

*Figure n° 1: Three Types of Development Paths*

Sustainable development paths
- Long-term increase in individual welfare
- Observation of certain safe minimum standards

Free market development paths

"Optimal" development paths

External effects

From the previous reasoning, we can also state the following..

*Proposition 2: Sustainability is not only an efficiency criterion; in addition to market efficiency, it is greatly concerned with the fairness between generations.*
Propositions 1 and 2 exhibit the approach to sustainability that is used below: a flow concept regarding long-term development of individual welfare. As noted, uncertainty can be incorporated in this concept, but it will not be a special focus of this paper. In accordance with mainstream economics, the analysis is tied to certain presuppositions. First, the analytical presentation in the neo-classical tradition relies on specific utility functions: it is assumed that preferences are individual and exogenous, that utility is derived from absolute not relative levels of consumption, and that the trade-off nature/consumption in individual valuation is a meaningful concept. Second, social aspects, i.e. the intragenerational income distribution and its impact on the environment, are not considered here. The emphasis is on dynamics, which is itself a broad and important field, as the discipline of environmental economics has largely remained in a static context so far.

Why should a society freely choose a sustainability constraint? A possible explanation is the "isolation paradox" (Marglin, 1963; Sen, 1967), which maintains that the collective concern for the future is greater than the private concern because of savings externalities. Thereby, the collective aim is not identical with government behaviour, as the government might have a time horizon that is even shorter than that of individuals. Why should a society rather not lower its discount rates? The reason is that the sustainability constraint is a more efficient way to reach sustainable development in no fewer than four different respects: (i) a very low or even zero discount rate may lead to investment rates that are so high that present generations cannot survive, (ii) a low discount rate harms future generations by increased pollution if capital accumulation has a pollutant effect which is not internalised, (iii) social discount rates reflect intertemporal shadow prices which depend upon the adopted numeraire so that the measure is quite vague, and (iv) capital accumulation and the decline in natural resources both influence the level of welfare and have to be weighed against each other. From a macroeconomic perspective, (iv) concentrates on the main point, on the well-being of future generations founded on basic macroeconomic relations. In the same way, Koopmans (1965, 1967) argues that we cannot have a direct intuition about the validity of discounting future well-being, unless we know something concrete about feasible development paths. Precisely this will form the content of the following sections.

3. A FLOW CONCEPT BASED ON ENDOGENOUS GROWTH

Turning back to the three types of paths summarised in figure n° 1, we can distinguish between three possible scenarios that are presented in figure n° 2. Scenario 1 shows the case in which the free market path and the optimal path are
both sustainable. Internalisation of external effects leads to higher welfare growth in the long term, which is itself a desirable target. In scenario 2, the free market path is not sustainable. But by internalisation of the external effects, with externalities being valued according to the preferences of current generations, sustainability is already achieved. This means that an environmental policy designed for current generations brings, at the same time, a development which is favourable for future generations.

The worst case is scenario 3, where both paths are not sustainable, i.e. even the optimal path does not meet the requirement for sustainability. Here, it is not sufficient to correct for all externalities; more stringent measures are necessary to obtain sustainable development. Which scenario materialises? The answer depends on the importance of natural resources in production and on the possibilities of substitution, to which I will turn now. The paths that are reached depend on production possibilities. Production relies on exhaustible and renewable natural inputs as well as on accumulable capital, such as physical, human, and knowledge capital. Pollution can be regarded as a special form of natural resource use which is --depending on the form of the pollution-- renewable or exhaustible. The pollution of the air, for example, decreases the stock of air quality. Some emissions such as NOx result in damages that are reversible, i.e. air quality is renewable, while other emissions such as FCKW have long-term consequences, so that the environmental service remains at a low level for a longer time, irrespective of counter-measures.

According to the Hotelling rule (Hotelling 1931), prices of exhaustible resources will, ceteris paribus, rise exponentially over time. This gradually lowers demand, i.e. the input of exhaustible resources into production. Changing monopoly power and the discovery of new stocks have, in many cases, prevented real developments from reflecting this prediction of theory. But rising prices should be expected in the future for at least some of the important natural resources and for pollution-intensive activities, where additional policy measures will most probably be implemented. Regarding renewable resources, harvest is sustainable if it equals natural regeneration. Harvest rates that exceed the natural regeneration rate lead to the extinction of the natural resource in the long run. In many cases, such as the world's fish stocks in the oceans, this happens because of the lack of property rights and free access externalities. The extinction of a renewable resource such as a certain species can be "optimal" from today's perspective, provided that the social discount rate is higher than the natural regeneration rate. But given some uncertainty about the future costs of extinction, the extinction cannot be assumed to be part of a sustainable development. In addition to the optimal use of exhaustible and renewable resources, which requires less natural inputs in the future, a reduction in natural resource use is also a political aim in
order to lower environmental risks. For economic theory to be used in this context this demonstrates the importance of substitution processes and the need to use well-founded theories to analyse this substitution. So, we conclude..

**Figure n° 2: Three Scenarios for the Development Paths**

![Three Scenarios for the Development Paths](image)

*Proposition 3: The conditions of the substitution of natural resources are the central topic of sustainable growth; macroeconomics provides useful tools to deal with the possibility of successful substitution.*

The second part of this proposition focuses on growth theory and especially on so-called new growth theory. It must be noted that neo-classical growth theory is not suited to the sustainability discussion, as the long-term growth rate is an exogenous variable in this model. What must be used is a theory that predicts endogenous growth in dependence on natural inputs. For this purpose, a growth mechanism has to be determined that relies –for reasons of microeconomic foundation– on the idea of positive spillovers. In accordance with this, let us state the following.
Proposition 4: A profound study of sustainable growth requires a sound theory of endogenous growth. To derive policy conclusions, it is necessary to focus on the analysis of paths that can be adequately described by theory.

This proposition means that, for policy purposes, there is little if no gain to compare hypothetical development paths that cannot be predicted by macroeconomic theory. What is predictable from existing growth theory are smooth development paths, whereas paths that consist of large deterministic "jumps" in individual welfare are not. The statement thus aims to limit the expected value of the discussion on the maximin criterion in the tradition of Rawls (1971), which implicitly assumes the existence of wide deterministic jumps. Given the existing possibilities of macroeconomic predictions, the Rawlsian approach thus boils down to the flow concept as used here. The only way to introduce rapid changes in welfare into growth theory is to introduce a stochastic process for the growth path, with possible interactions between the state of nature and the variance of this process.

In literature, Barbier/Markandya (1990) is a first contribution discussing sustainability in terms of formal growth theory. Gradus/Smulders (1993) cover a range of models of new growth theory including restrictions that stem from pollution, while Bovenberg/Smulders (1995) and Van Ewijk/Van Wijnbergen (1995) further explore the issue within two-sector models. For a textbook treatment of sustainability in terms of new growth theory, see Bretschger (1996a, chapter 10). The present paper extensively focuses on what is commonly assumed to be the most powerful growth engine: research and development. It adds substitution mechanisms that are relevant for sustainable growth but have not been highlighted in existing work yet. The results obtained in this paper rely on the fact of different input intensities of the sectors in an economy. They thus differ from Gradus/Smulders (1993), Bovenberg/Smulders (1995) and Van Ewijk/Van Wijnbergen (1995) who concentrate on the influence of natural resources on the efficiency in capital accumulation. One assumption there is e.g. that pollution lowers the ability of children to accumulate human capital which decreases economic growth. While this impact may be present in certain locations such as Mexico City, it seems that our analysis is broader and more general in both theory and practical relevance.

According to the sustainability concept introduced above, we regard the development of individual welfare under several production restrictions. The solutions calculated below correspond to the scenario of the free market path of section 2. In the multi-sector models introduced in section 5, positive externalities
are present so that the optimal path does not correspond to the free market solution. In all cases it will be discussed whether the path derived from the model is sustainable and, if not, which policy could lead to a sustainable development.

4. SUSTAINABLE DEVELOPMENT IN A ONE-SECTOR-MODEL

The easiest set-up to analyse how accumulated capital substitutes for natural inputs is the use of a one-sector model with accumulated capital and nature as inputs. Well-known earlier contributions in this tradition are Dasgupta/Heal (1974), Solow (1974), and Stiglitz (1974). A general production function in a one-sector economy is:

\[ Y = B \cdot F(K, N) \]  

(1)

where \( Y \) is aggregate output and \( B \) reflects the exogenous level of technology; \( K \) is aggregate capital which can be accumulated, such as physical, knowledge, and human capital. \( N \) is the natural input into current production, which can either stem from a renewable or from a non-renewable natural source. If the source is non-renewable, \( N \) will decrease over time according to standard resource economics; if it is renewable, \( N \) is assumed to decrease because of political measures (that are e.g. implemented to correct for the negative externalities of the natural resource use). From existing theory, it emerges that in a one-sector approach the elasticity of substitution between \( K \) and \( N \) in (1) plays the central role to analyse the consequences of a diminishing \( N \). As will become clear below, to reach sustainability this elasticity has to be unity or larger.

If the elasticity exceeds unity, nature is an inessential factor in the long run. At the same time, the income share needed for the compensation of the natural input steadily declines. This assumption on the value of the elasticity has been under severe critique with reference to the second law of thermodynamics. In this specific strand of literature, it is expressed that physical laws limit the extent to which physical capital can substitute for the natural capital stock, and especially for natural energy, as any transformation of matter requires a minimum of energy input. It has been argued that this minimum is no longer available from a certain point in time, because world-wide entropy steadily rises, which yields a drag for economic development. However, other authors have pointed out that the second law of thermodynamics only holds for closed systems, which does not apply to the planet earth. It has been confirmed that, in energy terms, the entire stock of natural resources on earth is not worth more than a few days of sunlight. Nordhaus (1992, p. 34) comes to the conclusion that “as long as the sun shines brightly on our planet, the appropriate estimate for the drag from increasing entropy is zero.”
Thus, physical laws of thermodynamics cannot be taken as having the consequence that the elasticity of substitution in relations such as (1) must unambiguously be at very low level. Nordhaus (1992) also argues that a macroeconomic formulation of the technical constraints such as (1) is sufficient to represent real production possibilities. With $N$ including energy inputs, there is, according to Nordhaus, no separate entropy constraint to be formulated in theory.

If the elasticity of substitution between $K$ and $N$ in (1) equals unity, which is a plausible value, sustainability entirely depends on savings behaviour. According to the well-known Hartwick rule, rents from exhaustible resources must be fully saved and invested in the accumulation of capital, so that the current income level can be infinitely sustained (Hartwick 1977). However, there has been some critique on this guideline in literature. It has been shown that the initial conditions of an economy may be such that the observation of the Hartwick rule is not sufficient, because the requested path may not be feasible. Also, if an economy follows an optimal path characterised by the maximisation of present value, where the Hartwick rule is more than satisfied, the level of consumption may begin to decrease after a certain time (for further explanation of these arguments see Toman/Pezzey/Krautkraemer, 1995). The most important point, however, is that savings depend on interest rates. If this relation is introduced, given (1) with an elasticity of unity, negative growth rates of income and welfare are very likely in the longer term. This happens because the Keynes-Ramsey rule, reflecting the intertemporal optimisation of households, predicts positive growth if the marginal product of capital exceeds the sum of the depreciation rate of capital, the population growth rate and the discount rate. But assuming constant returns to scale in (1), the marginal product of $K$ decreases with a declining input of $N$, so that growth rates become negative after a certain time period.

There are only two ways to invalidate this unfavourable prediction, while maintaining that savings depend on interest rates. First, one might assume an elasticity of intertemporal substitution that is close to zero, which, however, contradicts empirical evidence. Second, one can postulate increasing returns to capital $K$, which is empirically not well founded either (see e.g. Jones, 1995). It is only the constant return to capital $K$ that has been characterised as realistic in the research program of new growth theory. To motivate this assumption, one normally assumes the existence of positive spillovers and the term “capital” to include all macroeconomic assets that can be accumulated. It must be remembered that the traditional approach of economics is to assume decreasing returns to capital. The third case, assuming that the elasticity of substitution between $K$ and $N$ in (1) is smaller than unity, would imply the income share for the compensation of the natural input to steadily increase over time. This does not seem to correspond
However, the one-sector approach might be too strong a restriction. It is conceivable that, in an economy, there is not only a substitution of input factors, but also a substitution of industries that use nature with different intensities. Also, capital is not only an input into production, but also an output of certain industries. At any rate, one would like to evaluate the substitution process on a broader theoretical basis. It will turn out below that additional important substitution channels can be demonstrated in a multi-sector approach. From this we postulate:

**Proposition 5:** The use of one-sector models can be misleading concerning the conclusions on sustainability. It overstates the importance of the elasticity of substitution between natural and aggregate accumulated capital inputs and neglects the fact that capital is the output of specific sectors of an economy.

According to the Keynes-Ramsey rule, a central focus is the marginal product of capital which governs the growth process. In new growth theory, it is emphasised that a constant positive growth rate requires a marginal product of capital constantly exceeding the sum of the depreciation rate of capital, the population growth rate, and the discount rate. This can be achieved due to positive spillovers generated by capital investments. A constant return on capital yields a linear relationship between capital and income, or equivalently, the marginal product of capital is equal to the average product or average return on capital. Returns have to be valued with a general price index, whereas the capital good has to be valued with the price of capital. In the one-sector model used so far, these prices were identical, as the capital good is assumed to be produced with the same production technique as the final consumer goods. However, in reality, the difference between these prices is one of the key elements to determine the macroeconomic substitution process under consideration. To introduce different prices, a multi-sector model is required, with at least one sector where earnings accrue and one sector where capital goods are produced.

5. **SUSTAINABLE DEVELOPMENT IN THREE-SECTOR MODELS WITH R&D**

In the following, accumulated capital will be a sectoral output as well as an input for different sectors; among the accumulated capital factors, the emphasis
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will be on knowledge capital, while the other capital inputs are not introduced explicitly. An increase in the price of natural resources will influence the incentives for inputs to be used in different sectors, which may foster or hinder the substitution process under consideration; for a general treatment of the impact of sectoral changes on growth see also Bretschger (1996b). In the following model, two primary factors, labour and natural resources, are used for the production of consumer and capital goods. In the multi-sector approach, it is between these two primary factors that the substitution of inputs has to take place. This input substitution also governs the substitution between sectors, which obviously differs from the analysis in the one-sector model. As a consequence of a decreased input of natural resources into production, it is possible that more or less labour is devoted to capital investments entailing positive spillovers. In the first case, the growth rate of an economy increases, in the second case, growth decreases.

In general, it is realistic to assume that investment activities with positive spillovers are relatively more intensive in the use of labour, whereas the consumer sectors use relatively more natural inputs. So when natural resources become more scarce in production, capital investment can, under favourable conditions, be more attractive compared to the production of consumption goods due to changed relative prices. In this approach, it will turn out that a high elasticity of substitution between the primary factors may, while increasing the employment, harm the long-term growth rate. It will also emerge that the case of the elasticity being unity is again especially interesting, but one that leads to conclusions on sustainability that differ from the one-sector model.

Let us now turn to the formalisation of these mechanisms in this and the following two sections. We consider two versions of a three-sector endogenous growth model. In both versions, long-term growth depends on research and development (R&D) which constitutes one of the three sectors. The other two sectors are traditional goods $Z$ which are produced under constant returns to scale and high-technology consumer goods $Y$, which are assembled from intermediate inputs $x$ (see Romer 1990 and Grossman/Helpman 1991 for the basic model). Two kinds of increasing varieties forming the growth process are presented in two different model versions, which are called "model C" and "model K".

Figure n°3: Structure of Model C
In model C, the basic growth mechanism is a continuous expansion of varieties of \( x \)-goods in the \( Y \)-sector (the letter "C" applies because the expansion takes place in the consumer sector), see figure n° 3. In this figure, it can be seen that the primary inputs have to be allocated to the three sectors. Also, research and development is assumed to generate positive spillovers to knowledge capital, which is, in turn, an input into subsequent R&D (see corresponding arrows). The more knowledge is available, the more efficient is R&D, which keeps the development of new intermediates and the growth process going.

Model types similar to model C have often been used in growth literature; they are valuable versions of endogenous growth models. However, these models are restrictive in the sense that only the costs of design production count for long-term development, whereas the reward for inventions of new designs remains predetermined (see section 6 and Bretschger, 1996b). Also, long-term growth is a linear expansion of total consumption possibilities without any structural changes in production.

*Figure n° 4: Structure of Model K*
Primary inputs

- Natural Resource \( N \)
- Labour \( L \)
- High-Tech Capital \( R \)

Sectors

- Traditional Goods \( Z \)
- High-Tech Goods \( Y \)
- Intermediate goods \( x \)
- Research and Development
- Capital components \( q \)
- Public knowledge

But looking at significant changes in current production processes, such as technical progress in the car industry and housing, one notices that specialised high-tech capital has helped to reduce dramatically the need for natural inputs such as fossil fuels. For example, today it is possible to produce cars that use only a fraction of gas compared to the cars of two decades ago. In a similar way, the newest available materials have greatly contributed to the construction of houses that use a fraction of the energy for heating compared to older buildings. There can be no doubt that higher energy prices have acted as important incentives for these energy-saving innovations. It also becomes evident that high-tech capital and natural resources are good substitutes for each other and that high-tech capital has become cheaper relative to natural inputs due to technical progress.

These relations can be captured by model \( K \), which is presented in figure n° 4 and analysed in section 7. In addition to model \( C \), a second form of intermediate inputs \( q \) is introduced to produce an increasing variety of capital components that can be assembled to an aggregate capital input which I will call high-tech capital (see also Bretschger, 1996b and 1997a).

The following assumptions, which form the end of this section, apply for both model versions. Households are assumed to have homothetic preferences. They maximise a lifetime utility function, which is assumed to be additively separable in time and to contain logarithmic intratemporal subutilities of the Cobb-
Douglas type; \( \phi_Y \) and \( \phi_Z \) are the expenditure shares for the \( Y \)-good and the \( Z \)-good. It is convenient to choose the numeraire in such a way that the expenditures are normalised to unity at any point of time. R&D provides the know-how for the production of both types of intermediate inputs and is assumed to entail positive spillovers, leading to a constant growth rate. It is described as a production process with a natural input and labour as inputs, and so-called 'designs' containing the know-how to produce a new variety of differentiated goods as outputs. R&D is assumed to be the sector that is most intensive in labour, whereas the traditional sector is most intensive in the natural input, and the production of differentiated goods lies in between. Investors compare the profit obtained by developing a new design with the return on a second asset, which is assumed to be a bond. In the absence of differences in risk, the return on both assets must be the same in equilibrium. Then, the profit from an R&D-investment \( \pi \) plus the change in the market value of a design \( \Delta \text{v} \) is equal to the return \( r \) on an investment in bonds of equal size \( \text{v} \). This yields the equilibrium on the capital market as

\[
\pi + \Delta \text{v} = r \cdot \text{v}
\]

(2)

which holds at every moment in time. Let \( n \) denote the total number of designs. The intertemporal optimisation of households is given by the familiar Keynes-Ramsey rule. Stated in nominal terms and using the logarithmic form of the utility function, the growth rate of expenditures is determined by the difference between the nominal interest rate \( r \) and the discount rate \( \rho \). Choosing the normalisation of expenditures to unity leads (i) to an equality between the nominal interest rate and the discount rate, i.e. \( r = \rho \) and (ii) to total wealth being constant in long-term equilibrium, see Grossman/Helpman (1991, p. 60). As net aggregate wealth is equal to the total market value of all existing designs, i.e. \( n \cdot \text{v} \), a one percent increase in the number of designs (the growth rate of designs being denoted by \( g \)) is accompanied by a one percent decrease in the value of the designs, so that \( \dot{\text{v}}/\text{v} = -g \). Dividing (2) by \( \text{v} \) then gives for the capital market equilibrium:

\[
\frac{\pi}{\text{v}} = g + \rho
\]

(3)

In the following sections, 6 and 7, two different specifications of \( \pi \) and \( \text{v} \) in equation (3) will be derived. In section 7, the variable \( n \) will be replaced by \( m \) reflecting the different use of designs in the two models. Also, factor markets will slightly differ in model C from model K. In both models, \( Y \)-goods are assembled.
without cost from the existing variety of $x$-goods, which can be represented by a CES-assembly function (see appendix). A symmetrical equilibrium is regarded, in which all $x$s are of equal size and $X$ denotes total input in the $Y$-sector, i.e. $X = n \cdot x$.

6. SUSTAINABLE DEVELOPMENT IN MODEL C

The letter $a$ denotes the input requirement to produce one unit of a commodity, which is obtained by differentiating the unit cost functions with respect to factor prices $w$. $L$ stands for the labour supply, $N$ for the input of the natural resource. In model C, the equilibrium in factor markets is:

$$\begin{bmatrix} \bar{a}_{LZ}(w_N, w_L) \\ \bar{a}_{NZ}(w_N, w_L) \end{bmatrix} Z + \begin{bmatrix} \bar{a}_{LX}(w_N, w_L) \\ \bar{a}_{NX}(w_N, w_L) \end{bmatrix} X + \begin{bmatrix} \bar{a}_{Lg}(w_N, w_L) \\ \bar{a}_{Ng}(w_N, w_L) \end{bmatrix} g = \begin{bmatrix} L \\ N \end{bmatrix}$$ (4)

In model C, new designs are used for new intermediate inputs in the production of high-tech consumer goods. A firm that develops a new design earns a stream of profits that stems from marketing an additional intermediate good $x$ under the market structure of monopolistic competition (see appendix). This stream is infinite in time, as the assembly function for $Y$-goods is symmetrical (all designs are equal in value irrespective of the date of invention) and imitation of the design is assumed to be costly (so that only one supplier of a differentiated good acts on each market). In the CES-specification, the mark-up over marginal cost in the $x$-industries is constant. Here, it is denoted by $1 / \beta$, so that the profit $\pi$ for a design used in the production of intermediate goods $x$ is at each point of time:

$$\pi = (p_x - c_x(w_N, w_L)) \cdot x = (1 - \beta) \cdot p_x \cdot X/n$$ (5)

With free market entry into R&D, the profit per $x$-firm covers the development costs of the research lab. Research is assumed to raise the stock of general knowledge via positive externalities. General knowledge is, in turn, a free input into R&D. To use a simple scale, a one percent increase in knowledge is assumed to raise the design-output by one percent. Moreover, the stock of knowledge is equal to the number of designs $n$. Let $c_g$ denote the labour cost of generating one new design. To obtain the production cost for one new design, this
unit labour cost is divided by \( n \), so that the equilibrium value of a design \( v \) is

\[
v = c_g \left( \frac{w_g, w_L}{n} \right)
\]

(6)

Inserting (5) and (6) into the capital market equilibrium (3) and using the expenditure share for \( Y \)-goods yields:

\[
\frac{(1 - \beta) \cdot \phi_v}{c_g} = g + \rho
\]

(7)

The factor markets in (4) and the capital market equilibrium (7) form a system for three unknown variables, which are the two factor prices and the growth rate of designs. Without subsidies for research and development (generating positive externalities), it is given that an increase in the growth rate of designs means at the same time an increase in welfare, which we will assume. With the help of this system, the consequences of a decrease in the natural input can be calculated. To do so, one has to totally differentiate the three equations (see appendix for additional explanations). It then becomes possible to prove the following proposition:

**Proposition 6:** In the expansion-in-varieties in consumption model (model C), a decrease in the supply of the natural input harms the development of individual welfare, provided that the elasticities of substitution between labour and the natural input are larger than unity in both sectors of consumer goods.

**Proof:** The corresponding result of comparative dynamics is presented in equation (8).

\[
\hat{g} = \frac{1}{\Delta} \left[ \sum_{i'} \lambda_{i'v} \theta_{i'v} \left( \sigma_{v} - 1 \right) + \theta_{i'g} \left( \lambda_{i'g} \left( \sigma_{g} - 1 \right) + 1 \right) \right] \cdot \hat{N}
\]

(8)

\[
\Delta > 0 \quad i' = X, Z
\]

In (8), the \( \lambda \)s denote the factor shares and the \( \theta \)s are the cost shares; \( \sigma_{i'} \) is the elasticity of substitution between \( N \) and \( L \) in sector \( i' \) and hats denote growth rates. Given the factor-intensity rankings, the determinant \( \Delta \) is positive (see appendix). Furthermore, the terms under the summation sign on the right hand side of (8) are
positive if the elasticities of substitution between natural resources and labour are larger than unity as referred to in proposition 6. In this case, every decrease in \(N\) (as suggested by the theory of the previous sections) leads to a lower growth rate of the economy and thus decreases per-capita welfare; this means that the development path is non-sustainable. In contrast, low values for the elasticities of substitution between labour and nature can, according to (8), lead to sustainable development. In these cases, economic growth increases with a declining input of natural resources.

To get the intuition for the result, it is important to observe that the effect in (8) hinges entirely on the cost change in R&D, which is a weighted average of the factor price-changes for labour and natural resources. The reward for an innovation is predetermined in model C by use of the CES-function for \(Y\)-goods and the Cobb-Douglas utility function. If labour from traditional activities and the production of differentiated goods is released after the decrease in natural resources has taken place, wages decrease and the costs in the research labs fall. In contrast, if labour is attracted by the traditional or differentiated goods-sector, wages rise and the design-production is likely to become more expensive.

To find out which outcome materialises, one has to compare a substitution and an output effect. The substitution effect exhibiting the ability to substitute labour for natural resources is represented by the elasticity of substitution \(\sigma\). The output effects are the same for the two goods sectors. As expenditures for consumer goods are fixed, a one percent increase in the price of differentiated and traditional goods is accompanied by a one percent decrease in the quantity demanded. This is to say that the substitution effect is larger than the output effect if the elasticity of substitution exceeds the critical value of unity, which is the result of (8).

Assuming an elasticity of unity, it can be noted that the outcome in (8) corresponds to the well-known result of Romer (1990), if the factor "unskilled labour" of Romer is replaced by the natural input of our model, which is meaningful regarding the factor intensity ranking. Romer assumes that unskilled labour is not used in the research sector, which would mean, in this approach, that \(\theta_{Ng} = 0\). Moreover, he postulates a Cobb-Douglas production function, where \(\sigma\) is unity, as known from standard textbooks. He then obtains the result that the input of unskilled labour, which would be nature in our case, has no influence on the growth rate. This is exactly what must be expected from model C and can be verified by inserting \(\theta_{Ng} = 0\) and \(\sigma = 1\) in equation (8). From this it becomes obvious that the use of Cobb-Douglas production functions is a serious restriction, especially when studying the impact of factors that are not intensively used in
dynamic sector such as R&D.

7. SUSTAINABLE DEVELOPMENT IN MODEL K

The second version of the expansion-in-varieties model, model K, assumes varieties to serve as intermediate goods for a high-tech capital input as depicted in figure n° 4. In contrast, the number of \(x\)-varieties is assumed to be constant in this section. As will become clear below, it is now also the reward for designs that is significant for the growth rate; this reward will be dependent on the supply of natural resources. As in the case of \(Y\)-goods, high-tech capital goods are costlessly assembled from differentiated capital-components. The production of each component requires an initial design to be produced. In the general form, the quantity of this capital form denoted by \(R\) is given by a CES-assembly function which yields \(R\) as a function of the components \(q_j (j=1,\ldots,m)\):

\[
R = \left[ \prod_{j=1}^{m} (q_j) \right]^{1/\gamma} \quad \gamma > 1
\]

In growth theory, many authors assume the same technology for producing capital goods and final goods. Also, it is specific to high-tech capital that it is expensive to invent yet relatively cheap to produce. For a small theoretical model, it therefore seems useful to focus on the know-how spillovers from design production but not to introduce a separate sector for the production of capital goods. Therefore, let us assume that one of each capital-variety \(q\) is produced at zero cost. In this symmetrical case, the quantity of capital becomes:

\[
R = m^{1/\gamma} \quad (9')
\]

Regarding the empirical observations in the car and housing industries, it can be concluded that high-tech capital and nature are, in many cases, good substitutes for each other. In model K, high-tech capital is even assumed to be a perfect substitute for the natural resource. This strict assumption is introduced to set the opposite benchmark to model C. It has to be remembered that in these R&D models, we focus especially on the elasticity of substitution between nature and labour, which is in no way predetermined in model K. If the capital input \(R\) is a perfect substitute for the natural input \(N\), factor markets become:
The goods markets are the same as in section 6 (the number of x-varieties now being constant), but the incentives for R&D have to be derived somewhat differently. A firm that develops a design for a new capital component earns again a stream of profits that stems from marketing the additional component under the market structure of monopolistic competition. Similar to model C, this stream is infinite in time, as the assembly function for the R-factor is symmetrical and imitation of the design is assumed to be costly, as before. At any time, aggregate profits earned from designs for capital components are calculated by multiplying the quantity of capital \( R \) by the price of the natural input, i.e. \( w_N \cdot R \). These profits are equally distributed among the \( m \) designs for capital-components, which means for the reward of R&D given by \( \pi \):

\[
\pi = \frac{w_N \cdot R}{m}
\]  

(10)

With free entry into design production, the present value of the profit for a design is equal to the cost of design production. In this model version, the stock of knowledge is equal to \( m \exp(1-1/\gamma) \) to facilitate the calculations (\( \gamma > 1 \)). Again, the equilibrium value of a design \( v \) corresponds to the labour cost of producing one design divided by the stock of public knowledge:

\[
v = \frac{c_g}{m \exp(1-1/\gamma)}
\]  

(11)

Inserting (10) and (11) into the capital market equilibrium (3) and using (9') yields

\[
\frac{w_N}{c_g(w_N,w_L)} = g + \rho
\]  

(12)

Model K, reflecting an expansion-in-varieties of capital goods, is now used to examine the impacts of a decrease in the supply of natural resources on the growth rate. As in section 6, the two labour markets now being (4') are supplemented by the capital market equilibrium now being (12), which allows for
the endogenous determination of the factor prices and the growth rate of designs (see appendix). The second version of the model allows proof of the following proposition:

Proposition 7: In the expansion-in-varieties in production model (model K), a decrease in the supply of the natural input necessarily increases the steady-state rate of growth.

Proof: The corresponding result of comparative dynamics is presented in equation (13).

\[
\hat{g} = \frac{1}{\Delta'} \left[ (\lambda_{LX} + \lambda_{LZ}) (\theta_{NYS} - 1) \right] \hat{N}
\]

(13)

In model K, the effect of a decrease in the natural input on the costs of design production is still significant. But in addition, there is now an impact on the demand for high-tech capital, depending on the cost of the natural input. Regarding these two effects simultaneously, it turns out that the terms for factor substitution cancel each other out exactly, i.e. an additional unit of input does not change the factor-mix in production. This also means that the relative factor price is constant. What remains to be compared is the shift in total production costs (given the factor-mix) with the changing demand for capital. On the one side, the reward for developing a capital component is raised by a decrease in the supply of the natural input. On the other side, costs of design production are reduced. Taken together, the result can be interpreted as an expanded version of the well-known Rybczynski-Theorem. In (13), the negative relation becomes evident, as the cost share appearing in the second parenthesis is smaller than one by definition.

This means that in model K, inputs unambiguously move from consumer sectors to R&D after a decrease in \( N \), which raises the growth rate and welfare. Thus, the resource reallocation caused by rising prices of natural resources leads to a sectoral change in the economy that is favourable for sustainable growth.

8. SOME EXTENSIONS

After having presented two versions of a three-sector endogenous growth model, we can evaluate if a general prediction for an economy with more than three sectors is possible. In this case, there may be more than one sector where profits of capital accrue; also, there can be more than one sector where capital
goods are produced and spillovers are generated. Now suppose, realistically, that the learning intensive sectors, i.e. the sectors that generate a lot of positive spillovers such as R&D or education, are –on average– the sectors that do not use natural resources intensively. An increase in the price of natural resources relative to labour will then be efficient in obtaining sustainability in two ways. First, relative factor inputs will change in every sector. Second, there will be resource reallocation of inputs between sectors. Regarding growth by increasing varieties in the consumption sector (in analogy to model C), resource reallocation will be positive for long-term development provided that the elasticity of substitution between natural resources and labour is smaller than unity in each sector of the economy. Regarding growth by increasing varieties in the production sector (in analogy to model K), resource reallocation will be favourable for sustainability under all parameter constellations. The decisive impact of higher prices of natural resources is to decrease the relative weight of sectors that use the environment intensively, and to raise the importance of sectors that use relatively few natural resources. Also, endogenous knowledge production is most powerfully increased by the sectoral shift in an economy.

This argumentation, based on the multi-sector setting, relies on three presuppositions. First, it assumes the existence of positive spillovers, which has a long tradition in economics from Marshall (1920) to new growth theory. Second, it postulates different spillover intensities in the different sectors, which can be evidenced by empirical observations. Third, the hypothesis relies on a negative correlation between the spillover intensity and the natural resource use in the different sectors of an economy. If these three assumptions are valid, the relative importance of the different sectors emerges to be one or even to be the key element of the macroeconomic substitution process for sustainable development. From this we conclude..

Proposition 8: The change in relative prices has its major impact on sustainability by altering the sectoral structure of an economy.

For R&D and the growth rate that is based on innovations, it has been shown in model C that the lower the elasticities of substitution between the primary inputs are, the relatively lower the factor prices of non-natural inputs become and the faster economic growth is, as a consequence of higher prices of natural resources. This cost effect in the innovative sectors can be supplemented by higher incentives for nature-saving innovations, as shown in model K. It might be the case that an extended model introducing high-tech capital as an incomplete substitute for natural resources would allow further predictions here.

In open economies, changes in the international division of labour are an
additional factor to be observed when domestic factor prices are altered. The existence of "first mover advantages" rewarding countries in world markets that first introduce higher prices for natural resources is not unambiguous. What can be said in general, however, is that certain countries have a comparative advantage in providing environmental services. First, opportunity costs to conserve natural capital are different and second, some countries possess larger ecosystems of worldwide importance, so that the return on the conservation of the environment is comparatively large. The internationally joint implementation of measures for sustainable development, especially the changes in relative input prices, is a means to achieve the target with minimal economic costs. In this context, it can be expressed that..

**Proposition 9:** The international division of labour contributes to obtaining worldwide sustainability, and especially so, if prices for natural resources are on an level enabling sustainable growth worldwide.

Further applications of propositions 8 and 9 for open economies are to be found in Bretschger (1997a); for the problems to achieve sustainable development in open economies without international and interregional knowledge diffusion, see Bretschger (1998 and 1997b).

9. **CONCLUSIONS**

In the first part of the paper, it has been shown that the determination of endogenous growth paths with natural resources as inputs is a prerequisite to derive predictions on future development. Adopting a flow concept of sustainability, with safe minimum standards for specific natural resources, the substitution of labour and accumulable inputs for natural resources emerges to be the central focus of theory. In the second part, this paper provides additional information on the questions whether and how sustainable development is achieved. It demonstrates that increasing prices for the environment are the main impulse and that an economy's sectoral change is the main propagation mechanism to effectuate the substitution of natural inputs that is required for sustainability.

In the one-sector model, the results are more pessimistic regarding sustainability than in the multi-sector approach. It has been emphasised in the discussion of the one-sector model, that the assumption of an elasticity of substitution between capital and natural resources that is equal to unity, leads to an ever declining growth rate, i.e. to non-sustainable development. In contrast, in the multi-sector model, the assumption of an elasticity of substitution between labour and natural resources that is equal to unity, leads to a constant positive growth rate.
The second way to characterise real development moves the analysis to another position: It turns out that sustainability can indeed be achieved under realistic conditions. These conditions concern the elasticity of substitution in production and savings behaviour. Long-term growth is achieved by sectoral changes exploiting the allocative forces of market economies. This result is amplified by the impact of natural input prices on incentives for capital accumulation. If this link is introduced into endogenous growth theory, the prediction of a successful substitution process is even more likely to materialise.

To conclude, it should be remembered that the increase in prices of natural resources has to be effectuated by political measures if negative externalities of natural resource use are present. This has to be accomplished first. Also, and even more importantly, the sectoral change that is required for sustainable development has to take place at low economic costs. If adjustment costs are high, there is a drag in the growth process and the development path may become non-sustainable. The lower the cost of the reallocation in the direction of sectors that generate a lot of spillovers and do not use natural resources intensively, the better the chances for sustainable development. The political aim to lower adjustment costs is a better measure to achieve sustainable development than the general promotion of savings. This holds true because higher savings are not unambiguously in favour of sustainability, as long as some investments have a pollutant effect. In the international context, efficient prices of environmental services also lead to an income transfer from developed to less developed countries that increases the chances for LDC’s to achieve a sustainable development on their own. In addition to this international dimension, the risks of a low natural capital stock and several distribution issues should be a focus of future research.

**APPENDIX**

The high-tech consumer good $Y$ used in both model types is determined by the CES-form:

$$Y = \left[ \frac{1}{\beta} \sum_{j=1}^{n} (x_j)^{\beta} \right]^{1/\beta} \quad (0 < \beta < 1)$$

The elasticity of substitution between two intermediates is $\varepsilon=1/(1-\beta)$ and $1/\beta$. 


is the mark-up factor for every \( x \)-producer, assuming each individual producer is small compared to the rest of the market. In the symmetrical equilibrium, prices \( p \) of intermediates are identical, i.e. \( p_{xj} = p_x \) for all \( j \) and \( x \) becomes:

\[
x = \frac{(p_Y Y)}{n} = \frac{\phi_Y}{n}
\]

Standard price-cost relations are used to determine prices of \( Z \)-goods and \( x \)-goods from unit production cost that depend on factor prices (see also Bretschger 1996c). By inserting the budget shares \( \phi_Y \) and \( \phi_Z \), it then becomes possible to substitute for \( Z \) and \( X \) in factor markets (4). The differentiation of the labour market equilibrium in (4) yields:

\[
(\hat{a}_{LZ} + \hat{Z}) \cdot \lambda_{LZ} + (\hat{a}_{LX} + \hat{X}) \cdot \lambda_{LX} + (\hat{a}_{Lg} + \hat{g}) \cdot \lambda_{Lg} = \hat{L}
\]

with hats denoting growth rates. The \( \lambda \)s denote the fractions of the inputs used in each sector and the \( \theta \)s refer to the factor shares in each industry. \( \hat{g} \) stands for the percentage change in the growth rate of designs. We then have to insert \( \hat{a}_{Lz} = -\theta_{NZ} \sigma_X (\hat{w}_L - \hat{w}_N) \) and likewise for the other \( a \)-parameters, as well as:

\[
\hat{Z} = -\theta_{LZ} \cdot \hat{w}_L - \theta_{NZ} \cdot \hat{w}_N \text{ and } \hat{X} = -\theta_{LX} \cdot \hat{w}_L - \theta_{NX} \cdot \hat{w}_N
\]

For the natural input \( N \), the differentiation in (4) is analogous. Together with the differentiation of (7), one obtains the system for model C:

\[
\begin{bmatrix}
    b_{11} & b_{12} & \lambda_{Lg} \\
    b_{21} & b_{22} & \lambda_{Ng} \\
    \theta_{Lg} & \theta_{Ng} & \frac{\hat{g}}{g + p}
\end{bmatrix}
\begin{bmatrix}
    \hat{w}_L \\
    \hat{w}_N \\
    \hat{g}
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}
\]

The coefficients are:

\[
b_{11} = - \frac{\lambda_{Lz} \theta_{NZ} \sigma_X}{g + p} - \lambda_{Lz} \theta_{Lz}, \quad b_{12} = \frac{\lambda_{Lz} \theta_{NZ} \sigma_X}{g + p} - \lambda_{Lz} \theta_{NZ}
\]
\[
\begin{align*}
\mathbf{b}_{21} &= \lambda_{ni}\theta_{li}\sigma_i - \lambda_{ni}'\theta_{li}', \\
\mathbf{b}_{22} &= -\lambda_{ni}\theta_{li}\sigma_i - \lambda_{ni}'\theta_{ni}',
\end{align*}
\]

with \( i = X, Z, g \); \( i' = X, Z \)

The \( \sigma \)s represent the elasticities of substitution between labour and natural resources in each sector. In this system, the determinant of the left hand side is:

\[
\Delta = \frac{g}{g+\rho} (\mathbf{b}_{11}\mathbf{b}_{22} - \mathbf{b}_{12}\mathbf{b}_{21}) + \theta_{ng} (\lambda_{lg}\mathbf{b}_{21} - \lambda_{ng}\mathbf{b}_{11})
\]

\[
+ \theta_{lg} (\lambda_{ng}\mathbf{b}_{12} - \lambda_{lg}\mathbf{b}_{22})
\]

The value of the determinant is minimal if all the \( \sigma \)s are equal to zero. Then, it is:

\[
\Delta = \frac{g}{g+\rho} (\theta_{nx}\theta_{lx} - \theta_{nx}\theta_{lz}) \cdot (\lambda_{nx}\lambda_{lx} - \lambda_{nx}\lambda_{nz})
\]

\[
+ (\theta_{ng}\theta_{lx} - \theta_{ng}\theta_{lz}) \cdot (\lambda_{ng}\lambda_{lx} - \lambda_{ng}\lambda_{nx})
\]

\[
+ (\lambda_{ng}\lambda_{lz} - \lambda_{ng}\lambda_{nz}) \cdot (\theta_{ng}\theta_{lz} - \theta_{ng}\theta_{nz})
\]

With the assumed factor intensity rankings, the value of the determinant is positive as:

\( \theta_{nx} < \theta_{nz}, \theta_{lx} < \theta_{lz}, \lambda_{ng}/\lambda_{lg} < \lambda_{nx}/\lambda_{lx} \) and \( \lambda_{nx}/\lambda_{lx} < \lambda_{nz}/\lambda_{lz} \).

The system to calculate the comparative dynamics in model K is formed analogously from factor markets \((4')\) and the adjusted capital market \((12)\). After differentiating, one obtains:
\[
\begin{bmatrix}
\hat{b}_{11} & \hat{b}_{12} & \lambda_L \hat{g} \\
\hat{b}_{21} & \hat{b}_{22} & \lambda_N - \lambda_{23} \\
0 & \theta_{N}^{-1} & 0
\end{bmatrix}
\begin{bmatrix}
\hat{w}_L \\
\hat{w}_N \\
\hat{g}
\end{bmatrix}
= \begin{bmatrix}
0 \\
N \\
0
\end{bmatrix}
\]

with
\[
\hat{b}_{23} = \frac{R}{\gamma \cdot (N + R)} > 0
\]

In the system of model K, the determinant of the left hand side is:
\[
\Delta' = \frac{g}{g + \rho} \left( \hat{b}_{11} \hat{b}_{22} - \hat{b}_{12} \hat{b}_{21} \right) + \theta \hat{g} \left( \lambda_{N} \hat{b}_{23} - \lambda_{L} \hat{b}_{22} \right)
\]
\[
+ (\hat{b}_{12} \hat{b}_{23} - (\hat{b}_{N} - \hat{b}_{23}) \hat{b}_{11})
\]

Again, the value of the determinant is minimal if all the \(\sigma\)s are equal to zero. Here, it is:
\[
\Delta' = \Delta + 2 \cdot \lambda_2 \left( \lambda_{N} \theta_{N} \lambda_{L} \theta_{L} + \lambda_{N} \theta_{N} \lambda_{L} \theta_{L} \right)
\]
\[
+ \hat{b}_{23} \theta \left( \lambda_{N} \theta_{N} \lambda_{L} \theta_{L} + \lambda_{N} \theta_{N} \lambda_{L} \theta_{L} \right)
\]
\[
+ \hat{b}^* \left( \lambda_{N} \theta_{N} \lambda_{L} \theta_{L} + \lambda_{N} \theta_{N} \lambda_{L} \theta_{L} \right)
\]

with \(b^* = b_{13}(1 - \theta_{N}) - \lambda_{N}\). This expression is also positive given \(\Delta > 0\).

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**Résumé**

*Cet article propose une analyse macroéconomique du développement durable. Après avoir précisé le concept de sustainability, la nature des interdépendances entre ressources naturelles et accumulation du capital physique, humain et du savoir est présentée. Les conditions pour qu’un processus alternatif conduise au développement durable sont établies dans une approche mono-*
sectorielle et à travers deux versions d'un modèle de croissance endogène multi-sectoriel. Il apparaît que les prix des inputs naturels ont une influence majeure sur la croissance en modifiant la structure sectorielle de production. L'hypothèse d'un remplacement efficace des ressources naturelles avec les progrès de la connaissance est plausible, si les costs d'ajustement sectoriel dans l'économie ne sont pas trop élevés.

Resumen

Este artículo propone un análisis macroeconómico del desarrollo duradero. Después de precisar el concepto de sustainability se presenta la naturaleza de las interdependencias entre recursos naturales y acumulación del capital físico, humano y del conocimiento. Las condiciones para que un proceso alternativo conduzca al desarrollo duradero se establecen en un estudio monosectorial y a través de dos versiones de un modelo de crecimiento endógeno multisectorial. Notamos que los precios de los insumos naturales tienen una influencia importante en el crecimiento modificando la estructura sectorial de producción. La hipótesis de sustitución eficaz de los recursos naturales con los progresos del conocimiento es verosímil, si los costes de ajuste sectoral en la economía no son demasiado altos.