ON POTENTIALIZED PARTIAL FINITE DIFFERENCE EQUATIONS: ANALYZING THE COMPLEXITY OF KNOWLEDGE-BASED SPATIAL ECONOMIC DEVELOPMENTS

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Abstract - *Knowledge-based regional and urban studies are plentiful; some systematics might be in order at this junction, so first the different links between economic production units in geographical space have to be clearly defined. Then a tool to represent the dynamics of those links should be selected; potentialized partial differential equations (PPDEs) are an appropriate tool to represent space-time relations in pre-geographical space. In practice, however, only discrete data are available, hence the question of how finite differences could generate PPFDEs (potentialized partial finite difference equations). A case has been worked out and simulated, showing a high degree of spatio-temporal complexity. Spatial econometric estimation is possible, as other work has shown; so an application to empirical data for France could be presented. Different versions of the latter have been worked out; they are presented in succession, followed by a last exercise on US data.*

Keywords: COMPLEXITY, SPATIAL ECONOMETRICS, POTENTIAL, FINITE DIFFERENCES

JEL Classification: F13, F15, Q17.

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1. INTRODUCTION

Numerous studies have been devoted to knowledge-based economic activities, especially in geographical space (see e.g. Acs, 2000; Acs et al., 2002;Aygerou and La Rovere, 2003; Bröcker et al., 2003; Christensen and Maskell, 2003; Egidi and Rizzello, 2003; Fisher et al., 2001; Geuna et al., 2003; Hemlin et al., 2004; Lagnevik et al., 2003; Maggioni, 2002; Maier and sedlacek, 2005; Nahuis, 2003; Rodriguez, 2003; Schätzel and Revilla Diez, 2002)); as links between located producing units are essential for the workings of such a spatial economy (see Leoncini and Montresor, 2003), those links have to be clearly specified. In Paelinck (2004c) a systematic classification of those links has been given; its main elements will be taken up again.

In previous studies (Kaashoek and Paelinck, 1994, 1996, 1998 and 2001) different aspects of potentialized partial differential equations (PPDEs), as analytical tools for studying space-time relations over pre-geographical space, have been studied. They will be shortly taken up in section 3, section 4 studying the possibility of switching to PPFDEs, potentialized partial finite difference equations.

This possibility having been shown, a simulation study is reported; section 6 treats the estimation problem with empirical applications to France, section 9 using US data.

Conclusions and references appear in sections 10 and 11, respectively.

2. SPATIAL ECONOMIC LINKAGES

Paelinck (2004c) presents five concepts ("the 5 Cs") to clarify the sometimes foggy terminology used in some of the studies on spatially intertwined economic activities

The first one is that of a *cluster*, which can be defined as a set of located economic activities (production plants, service outlets, a.o.), such that for every unit one can find one or more activity units lying within a given distance d^* from the unit considered; a cluster is thus a purely geographical concept.

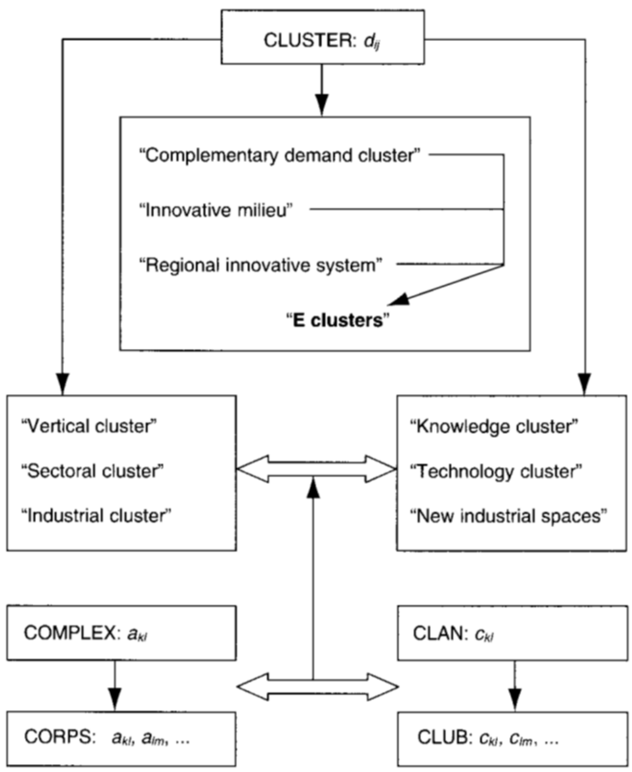
The next idea needs the introduction of a techno-economic relationship, to wit an input coefficient, defined as the share of a product or service , needed in the production of a given item, in that item's total production value; this allows to define a *complex* as a cluster with an uninterrupted chain of input relations between the set of its activities (in mathematical terms, this means that the corresponding input matrix is indecomposable),

A third concept is that of a *corps*; it is defined as a sub-complex that maximizes a partial sum of relevant activity flows, still leading up to a non-decomposable set of activities (Paelinck, 2002); in that sense a complex can be made up from several corps, possibly in declining order of internal cohesion, measured by the above-mentioned sum.

Finally, two new notions are introduced: *clans* – which transpose the idea of a complex to "contact coefficients" –, and *clubs*, which generalize the idea of a corps to the same coefficients.

Figure 1 hereafter (from Paelinck, 2004c) shows that mixed concentrations can occur (see the two-way arrows), so the idea is to identify the relative strength of the locational forces at work; how complex they can be is well illustrated in Berger (2006), a fact which is particularly true of knowledge-based activities.

Figure n°1 : Conceptual flow diagram for 5C analysis



In order to start up a dynamic analysis of those complexities, a tool is needed that could represent them, hence next sections on one of those possibilities.

3. PPDEs

Space-time representations have since long been expressed in terms of partial differential equations; considering only one space variable *x* and time *t*, a partial differential equation – abbreviated as PDE – for some function *g(x,t)* is a relation of the form:

$$h(x,t;\ g_x,g_t;\ g_{xx},g_{xt},g_{tt};...) = 0 \tag{1}$$

where, in general, *h* is a given function of the independent variables, *x* and *t*, of the still unknown function *g*, and of a finite number of its partial derivatives.

Recall the wave equation :

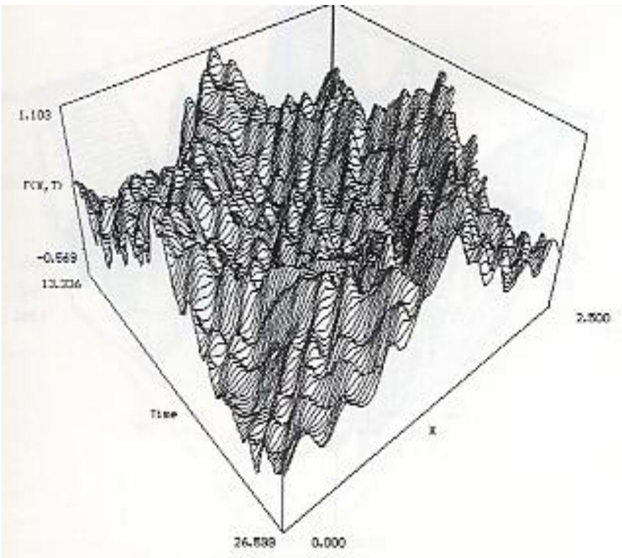
$$\ddot{f}(x,t) = \alpha^2 f''(x,t) \tag{2}$$

the double dot meaning the second time-derivative (acceleration, if one likes), the double prime the second *x*-derivative. Equation (2), as many other ones commonly studied, especially in theoretical (non-quantum) physics, is an expression of *local* interaction, but in spatial economics, as in quantum physics, "non-locality" is the rule, so to express spatial interaction, equation (2) should be generalized to :

$$\ddot{f}(x,t) = \alpha^2 \int_{-l}^{+l} w(x,\xi) f''(\xi,t)\, d\xi \tag{3}$$

where $w(x,\xi)$ is a so-called "spatial discount function", its convolution with some variable representing a potential over a line [-l, +l]; hence equation (3) can be called a PPDE, a potentialized partial differential equation.

Figure n°2 : Realization of a PPDE



Kaashoek and Paelinck (see references in section 1) have studied various aspects of PPDE's, i.a. two-dimensional spatial cases, the effects of varying the potentializing function, and the possibility of controlling the space-time process; the latter problem crept up from the fact that many realizations of that process happened to be chaotic, but as it is generated from "exact" equations, it belongs to the family of so-called "exact" chaotic processes.

Figure 2 pictures one such process.

One will notice the presence of sharp "peaks" which have been dubbed "pseudo-solitons", as genuine solitons are in fact infinitely dense local peaks (Dirac functions), but pseudo-solitons, like the true ones, can travel over space, as figure 2 clearly shows.

4. PPFDEs

Rewriting equation (3) in a finite difference specification is an easy task which results in :

$$\Delta^2_t f(x,t) = \alpha^2 \Sigma_{-1}^{+1} w(x,\xi) \Delta^2_x f(x,\xi) \quad (4)$$

the summation depending on the spatial interaction process selected.

In what follows that process will be stopped after a first degree contiguity effect; to illustrate this an example with eleven units located on a line (see figure 3) will be further developed.

Figure n°3 : Eleven spatial units on a line

• a • b • c • d • e • f • g • h • i • j • k •

The set-up chosen was, as said, a first degree contiguity order next to a local spatial second finite difference effect, the latter having a reaction coefficient of .5, the neighboring effect being of the order of .25. The initial two time patterns were zero for all the units except the central one which was perturbed to 1, and its neighbors, perturbed to .5; the relevant outside spatial units were systematically allotted a zero-level.

This resulted in the following specification:

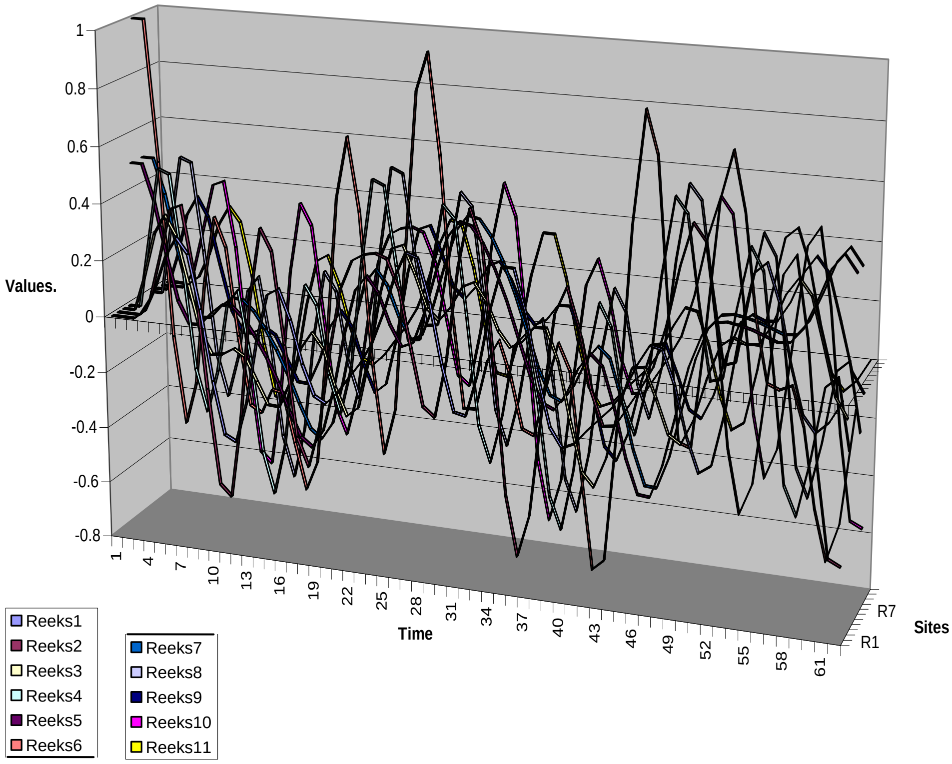
$$\Delta^2_t (x,t) = .5\Delta^2_x(x,t) + .25\Delta^2_x(x-1, t) + .25\Delta^2_x(x+1, t) \quad (5)$$

due corrections having been made in the case of border regions.

Figure 4 hereafter reproduces a simulation over 62 periods; the simulation has been performed according to the following expression for a fully-fledged effect (unit *c*, e.g.), the indices of the spatial units referring to time-periods :

$$c_3 = 2c_2 - c_1 + .5(b_2 - 2c_2 + d_2) + .5(a_2 - 2b_2 + 4c_2 - 2d_2 + e_2) \quad (6)$$

Figure n°4 : Simulation of a PPFDE



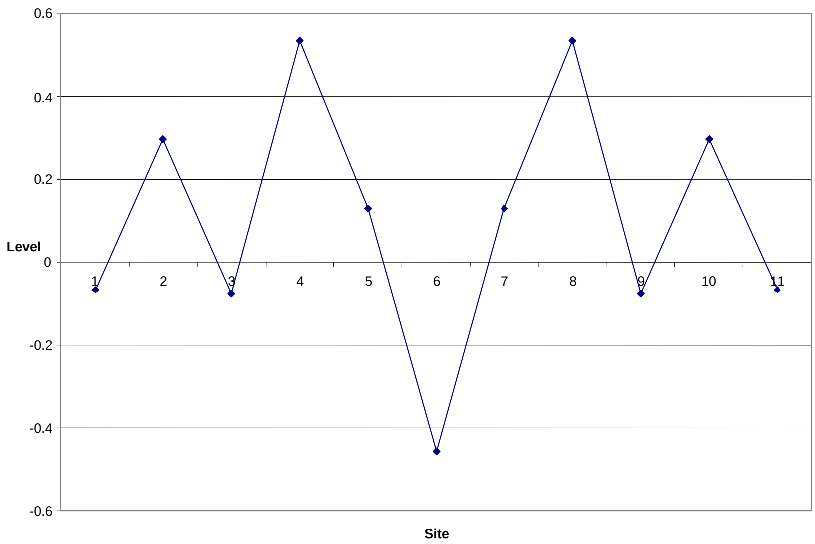
The graph has all the characteristics of figure 1, and illustrates the fact that simple processes, like the one of equations (5) or (6), can produce very complex patterns (Frenken, 2004 ; Paelinck, 2000 ; Quadrio Curzio and Fortis, 2002 ; Rosser, 2004 ; Wolfram, 2002).

Figure 5 is a cross-section at one point of time.

From the obtained simulation results, a sample of 10 consecutive time-slices has been extracted, which allowed of computing the implicit parameters using equation (6); the estimated - by OLS - parameters came out identical to the values used for the simulation. .

This is indeed what spatial econometrics is about; purely theoretical approaches are most valuable (Paelinck, 1997; Zhang, 2022), but testing the relationships discovered that way is indispensable to empirical knowledge. A first effort in this sense has been applied to French data, which will be described hereafter, after which the estimation results will be presented.

Figure n°5 : A typical time-slice of figure 4



5. APPLICATION TO FRENCH DATA

The development of knowledge-based industries will be modeled for the second most populated region in France after Ile-de-France, i.e. the Rhône-Alpes region. That modeling will be based on the following data:

- employment in activities close to the concept of knowledge-based industries for at least 3 periods,
- in the main towns of Rhône-Alpes;
- and also distances between those towns.

5.1. Employment

Since 1990, Philippe Julien (1995, 2002) from INSEE (National Institute for Statistics and Economic Studies) has overcome the over-simplified nature of sectoral employment and of PCS (classification of professions and socio-professional categories) to identify what he named the « strategic employment of French towns ». He then called this new statistical category: « upper metropolitan functions » of which the role was *a priori* the most dynamic for the activity level.

The « upper metropolitan functions » are composed of eleven functions resulting from a crossing-over between the sectors of activities and the professions represented by the higher level of skills, mainly executive and engineers. Public and intermediate functions have been excluded.

Between 1990 and 1999, the share of upper metropolitan employment has increased from 8,2 % to 9,0 % of total employment in the 354 “metropolitan areas” ("aires urbaines"), the urban level chosen for the modelling.

The three periods considered are 1982, 1990 and 1999. The figures for 1990 and 1999 are observed data based on French censuses. Data for 1982 are estimated by a linear model taking into account the share of executive and engineers in 1982 and the difference between the share of executives and engineers and the share of “upper metropolitan employment” in 1990. The share of executives and engineers has increased by 37 % between 1982 and 1990, which explains the strong growth of the estimated “upper metropolitan employment” during this period.

5.2. Main “urban areas” of Rhône-Alpes

The concept of “urban areas” is defined by INSEE as a continuous urban region made up of a core (statistical urban unit of INSEE) and a crown of municipalities of which the commuters depend on the core for at least 40 %.

Thirty-seven “urban areas” are so considered, the largest ones being those of Lyon, Grenoble and Saint-Etienne (table 1 and map 1).

5.3. Distances and travel times by road between “urban areas”

Distances (table 2) have been calculated from city-core to city-core: as the crow flies, by road kilometers, and by average travel time by road; this last definition has been chosen for computation.

6. FIRST ESTIMATION RESULTS

From the topographical data commented on in section 5, the two nearest neighbors of 37 urban regions have first been selected.

As differential growth variable, one has selected a yearly average over 1990-1999, minus the same average over 1982-1990.

Then the second differentials have been calculated over the nearest neighbor set, first for the urban region in question, and the for its selected two neighbors; this then leads to three explanatory indices, noted n_{0i}, n_{1i} and n_{2i}, standing for neighborhood degrees 0, 1 and 2 respectively.

Ordinary least squares was used to obtain the regression parameters of the equation :

$$\Delta^2_{ti} = a\Delta^2 n_{0i} + b\Delta^2 n_{1i} + c\Delta^2 n_{3i} \qquad (7)$$

Other methods of estimation being envisaged in further analyses (Paelinck, 2004a).

The estimation results appear in table 3.

Table n°1 : Upper urban employment of the 37 urban areas in 1982, 1990 an 1999

Urban areas	*1982*	*1990*	*1999*	*EmpTot99*	*EmpStrat82*	*EmpStrat90*	*EmpStrat99*
LYON	43 364	65004	75935	714469	43 364	65004	75935
GRENOBLE	15 891	22612	28202	221851	15 891	22612	28202
SAINT-ETIENNE	5 284	7228	8084	128582	5 284	7228	8084
GENEVE(CH)-ANNEMASSE (French part)	981	2640	2933	56791	981	2640	2933
ANNECY	3 744	5896	6975	84087	3 744	5896	6975
VALENCE	3 033	4328	5008	71131	3 033	4328	5008
CHAMBERY	2 656	3520	4149	58937	2 656	3520	4149
ROANNE	1 440	1664	1688	42640	1 440	1664	1688
SAINT-CHAMOND	875	996	1192	26106	875	996	1192
THONON-LES-BAINS	690	1036	1037	24538	690	1036	1037
ROMANS-SUR-ISERE	689	816	1114	24620	689	816	1114
VILLEFRANCHE-SUR-SAONE	1 038	1408	1602	27380	1 038	1408	1602
CLUSES	925	1392	1626	34382	925	1392	1626
MONTELIMAR	638	840	955	22814	638	840	955
VIENNE	672	932	1130	21966	672	932	1130
SAINT-JUST-SAINT-RAMBERT	443	688	846	17489	443	688	846
VOIRON	395	636	848	15481	395	636	848
SALLANCHES	317	528	604	17268	317	528	604
AUBENAS	365	536	445	14513	365	536	445
AIX-LES-BAINS	522	780	855	14135	522	780	855
ANNONAY	328	464	603	16336	328	464	603
ROUSSILLON	362	504	430	13684	362	504	430
ALBERTVILLE	274	472	471	12811	274	472	471
BOURGOIN-JALLIEU	497	688	735	15466	497	688	735
MONTBRISON	242	368	407	10172	242	368	407
PRIVAS	249	352	374	9662	249	352	374
TOURNON-SUR-RHONE	204	332	270	7650	204	332	270
TARARE	213	244	343	7688	213	244	343
LIVRON-SUR-DROME	98	136	130	5388	98	136	130
RUMILLY	146	204	333	7549	146	204	333
SAINT-MARCELLIN	172	192	197	6012	172	192	197
CHAMONIX-MONT-BLANC	91	160	192	6668	91	160	192
SAINT-JEAN-DE-MAURIENNE	118	144	239	6215	118	144	239
TOUR-DU-PIN	116	152	223	5591	116	152	223
PIERRELATTE	539	740	595	8833	539	740	595
FEURS	167	192	176	5857	167	192	176
BOURG-SAINT-MAURICE	85	132	134	5401	85	132	134

Table n°2. Examples of distances

area to area	*as crow flies*	*Km by road*
Lyon - Grenoble	37	69
Lyon - Chambéry	88	100
Lyon - Saint-Etienne	49	62

Map n°1 : Rhône-Alpes and" urban areas"

Table n°3 : First estimation results

Parameters	*Values*	*t- or F-statistic*	*Probability*
a	*-.0041*	*-3.1574*	*.0033*
b	*-.0049*	*-4.0414*	*.0003*
c	*.0002*	*.2927*	*.8174*
R^2	*.5156*	*12.0615*	*1.72E-05*

The values of the parameters appear to be small, but this is due to the relative levels of regressand and regressors; the important to notice is the extreme significance of *a* and *b*, and their negative signs, to be interpreted as (potentialized, for *b*) anti-wave behavior. The overall fit is also extremely significant, given the number of degrees of freedom (*34*). Only the second order effect is positive but non-significant. A regression with a constant was performed, but it appeared to be non-significant (p = .2965).

7. A FIRST SIMULATION OF THE MODEL, NEW ESTIMATION AND SIMULATION OF THE REVISED MODEL

Though the estimation results of the PPFDE are up to now the first ones obtained in this area, a first simulation has been carried out, in order to check on the behavior of the model. It is started from the years 1989-1990, the first one having been computed by retropolation of the average absolute growth over the years 1982-1990.

A large number of projections (8, exactly) turned out to become rapidly negative (we will come back to this point), due to the large values of the of the relevant spatial second differences (the explanatory variables). Deleting those terms resulted in only a few sites now showing negative values from more or less their half-way path.

It is interesting to compare observations and projections; figure 3 presents the second ones as a function of the first ones.

Its characteristics are the following (table 4).

Figure n°6 : Observations and projections 1999

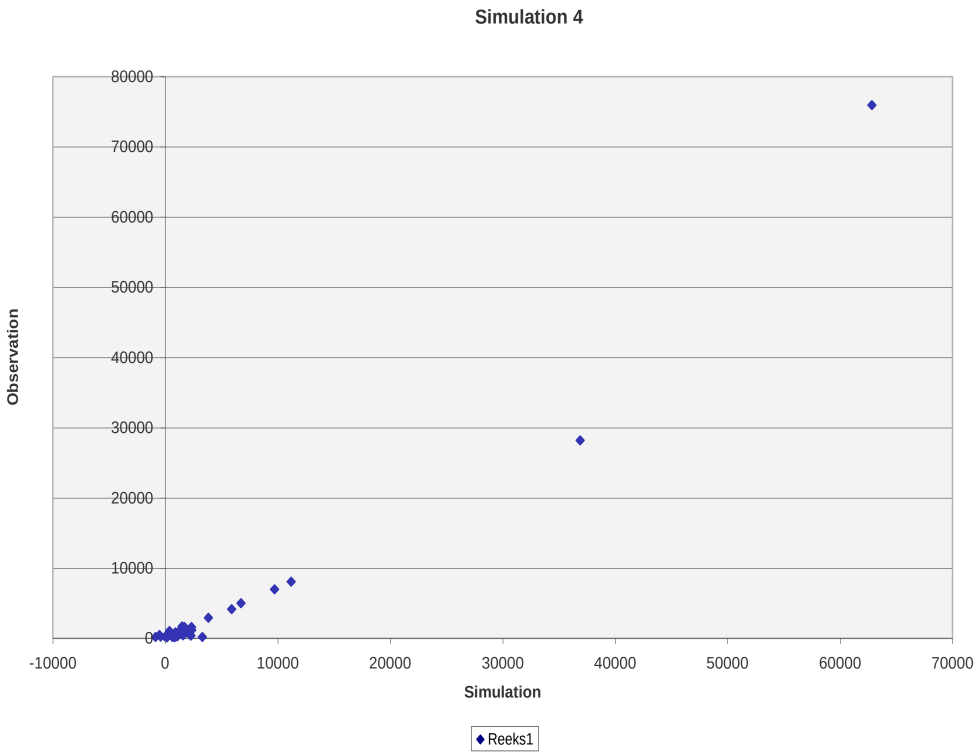


Table n°4 : Statistics of figure 6

Parameters	***Values***	***Test***	***Probability***
Constant	-942	t = -2.0260	.0504
Slope	1.0911	t = 29.1911	≈0
R^2	.9801	F =652	≈0
U(Theil)	.1543	σ=.0254	-

Without being perfect, these first results are for the least satisfactory; they suggest three possible improvements ;

- work in logarithms, which would dampen extreme values of the explanatory variables, avoiding at the same time negative values of the natural variables;
- introduce two regimes in the regressions (Paelinck, 2003b);
- work in two dimensions (Kaashoek and Paelinck, 1998).

These suggestions will be treated in what follows.

As the simulations reported suggest, there is evidence for the existence of at least two regimes; the equation to be estimated is now the following:

$$\Delta^2\ln(n_{t0i})=\lambda(a\Delta\ln(n_{0i})+b\Delta\ln(n_{1i})+c\Delta\ln(n_{3i}))+(1-\lambda)(a^*\Delta\ln(n_{0i})+b^*\Delta\ln(n_{1i})+c^*\Delta\ln(n_{3i})) \quad (8)$$

the λs being binary switching variables; the model has been estimated by minimizing absolute discrepancies to neutralize outliers. Table 5 presents the results.

Table n°5 : New estimation results

Parameters	***Values***
a	.012176
b	.007219
c	-.001544
a*	-.005856
b*	-.000270
c*	.000829
Pseudo-R^2	.9990

The result is remarkable: the regimes are each other's reverse in terms of signs, and the fit is almost an interpolation, so all coefficients should be significant; table 6 pictures the regimes.

The numbering of the sites does not correspond to that of table 1, but is alphabetical. It can be checked that smaller sites in general belong to regime 1 ($\lambda = 1$), larger agglomerations (Grenoble, Lyon, Saint-Etienne, e.g.) to the other one.

As to the simulation, it continued to underestimate the values for 1999, so two corrections were made: a general dampening factor for all parameters, and an autonomous increase for all functions. They were computed so as to reproduce the total for 1999, and had values respectively of .166703 and .003949.

Table 7 replicates table 4, but now in terms of the new simulation which is finally graphically presented by figure 7.

Table n°6 : Regimes

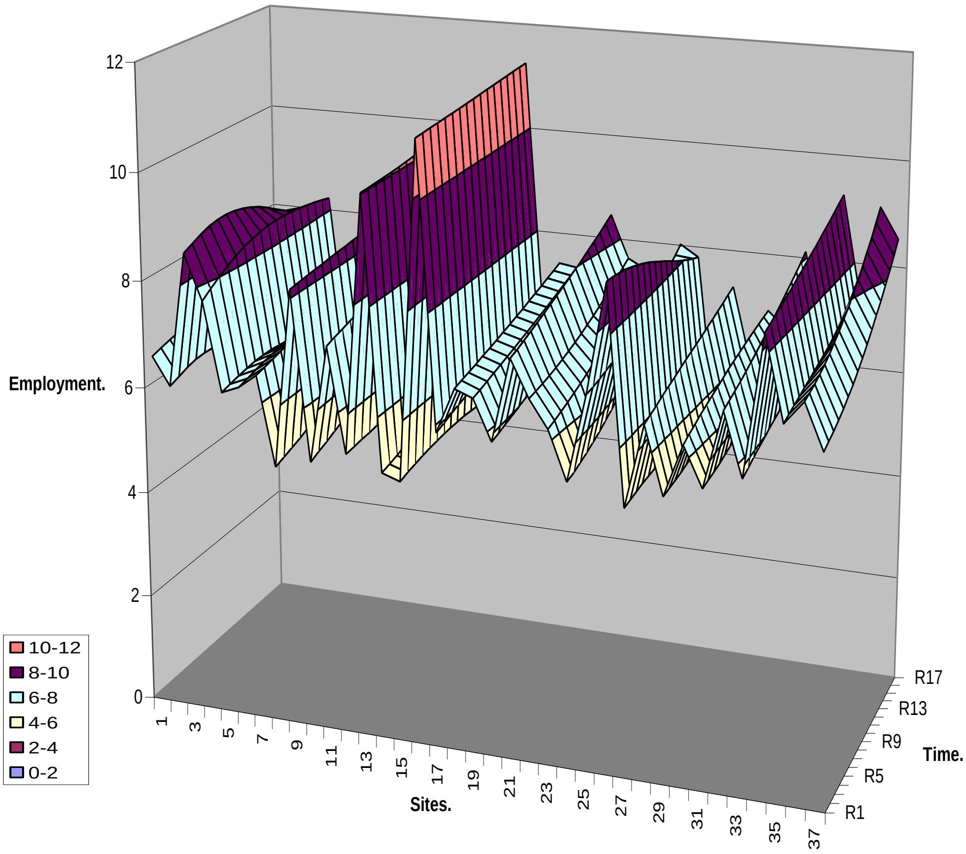
Sites	***λ-values***	***Sites***	***λ-values***
1	1	20	1
2	0	21	0
3	0	22	1
4	0	23	0
5	1	24	0
6	0	25	0
7	0	26	0
8	0	27	1
9	0	28	1
10	1	29	0
11	1	30	1
12	1	31	1
13	0	32	1
14	1	33	1
15	1	34	0
16	0	35	0
17	1	36	0
18	1	37	0
19	1		

Table n°7 : Statistics of the second simulation

Parameters	***Values***	***Test***	***Probability***
Constant	-256	t = -1.4898	.1452
Slope	1.0627	t = 78.7929	≈0
R^2	.99944	F =6208	≈0
U(Theil)	.0660	σ=.0109	-

The figures show indeed a closer fit between observations and projections. If the latter are to be favored, a possible solution would be to interpolate quadratically the 1982-1990-1999 observations, and use equation (6) for estimation purposes; anyway we now know that at least two regimes are present. Be it noted that the above "dampening-cum-acceleration" correction might have been necessary, due to the fact that an average growth figure for the period 1982-1990 was used, whereas growth had considerably decreased over the next period (the total figures are respectively 87,863; 128,956 and 151,080).

Figure n°7 : Second simulation



8. ESTIMATION FOR SIMULATION

The estimations of section 7 have been performed locally to test the model; if one wants to use it for simulation or projection purposes, a time series should be available. Such a series has been constructed by quadratic interpolation using the logarithms of the three available observations.

Table 8 presents all the estimation results obtained, A being the first one of section 6 (table 3), B the second one (table 5), C and D new ones, respectively computed by OLS and LAD ("least absolute discrepancies") using equation (6) over all the sites.

Table n°8 : All regression results

Parameters	*A*	*B*	*C*	*D*
a	-.0041	.012176	.000867	.000430
b	-.0049	.007219	-.000949	-.001148
c	.0002	-.001544	-.000108	2E-05
a*	-	-.005856	-.000248	-.000122
b*	-	-.000270	-.000113	-9.88E-05
c*	-	.000829	3.5E-05	-4.81E-06
(Pseudo)R^2	.5156	.9990	.9571	.9560

One has to remember that A has been computed from natural values; comparing B with C and D one notices a clear dampening effect, already quoted before and after table 7.

Table 9 lists the λs as did table 6.

Table n°9 : New regimes

Sires	*λ-values*	*Sires*	*λ-values*
1	1	20	0
2	0	21	0
3	0	22	0
4	0	23	1
5	0	24	0
6	0	25	0
7	0	26	0
8	0	27	0
9	0	28	1
10	0	29	0
11	0	30	1
12	0	31	1
13	0	32	1
14	0	33	1
15	1	34	0
16	0	35	0
17	0	36	0
18	0	37	0
19	0		

One notices a clear divergence with the previous results (table 6); the Hamming distance (Paelinck, 1983, pp. 44-50) $d_H = .3243$, corresponding to a Kendall rank correlation coefficient (Kendall, 1955) $\tau = .3514$. However Grenoble, Lyon and Saint-Etienne still belong to regime $\lambda = 0$.

9. BI-DIMENSIONAL TREATMENT

It is well-known (see Braun, 1975, p. 684) that the generalization of partial differential equations to bi-dimensional space can be realized through a Laplace-operator; take again equation (2), which then turns to :

$$\ddot{f}(x,y,t) = \alpha^2 \nabla f(x,y,t) \tag{9}$$

with :

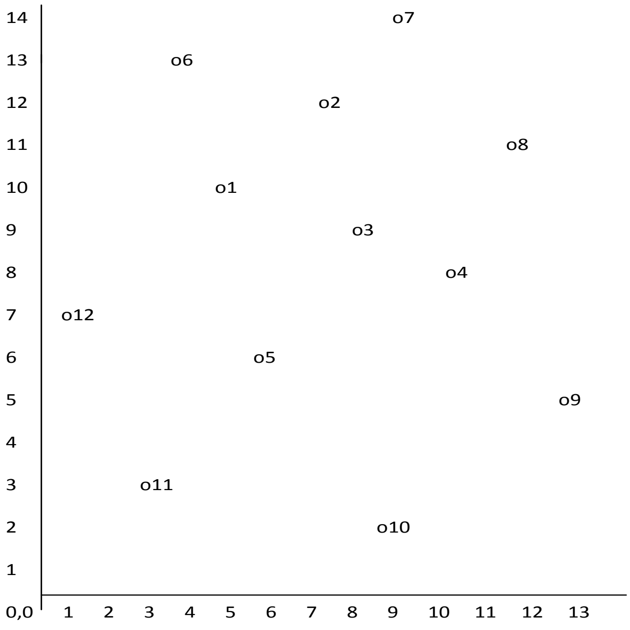
$$\nabla f(x,y,t) = \partial^2/\partial x^2 f(x,y,t) + \partial^2/\partial y^2 f(x,y,t) \tag{10}$$

the potentialized form then turning to :

$$\ddot{f} = \alpha^2 \int_\xi \int_\eta w(x,\xi;y,\eta)\nabla f(\xi,\eta,t)d\xi d\eta \tag{11}$$

Equation (11) is easily generalized to second finite differences, $\Delta^{2\prime}{}_x f$ and $\Delta^{2\prime}{}_y f$.

Map n°2: Locations of the relevant points



The empirical problem consists in being able to compute these second differences around the relevant points. In the fictional example hereafter additional points have been constructed so as to enable that operation; in fact this means that an interior sample was used for the computations.

Map 2 presents the points, the following tables 10 and 11 the numerical data, sites 6 though 12 being auxiliary.

Table n°10 : Rounded-off distances

Sites	1	2	3	4	5	6	7	8	9	10	11	12
1		2	3	6	5	2	7	7	10	9	9	4
2			3	6	6	3	3	5	9	10	10	6
3				3	4	5	6	6	8	7	8	7
4					5	8	5	3	4	7	10	10
5						8	9	9	7	5	8	4
6							7	8	11	14	12	7
7								4	12	13	15	14
8									6	11	14	13
9										6	12	12
10											7	10
11												13
12												

Table n°11 : Function values

Sites	f_{t-1}	f_t	f_{t+1}	$\Delta^{2'}{}_t$
1	10	12	11	-3
2	5	7	10	1
3	8	6	9	5
4	3	5	2	-5
5	2	1	3	3
6		4		
7		2		
8		3		
9		1		
10		5		
11		6		
12		4		

Table 12 reports the relevant values for the calculation (by OLS); the first column shows the own Lagrange operator, the second column the Lagrange operator of the nearest neighbor, the third column the second time difference *x10*, the regressand.

Table n°12 : Regressors and regressand.

∇	∇^*	$\Delta^{2,}{}_{t}$
-20	5/6	-30
5/6	-20	10
$6^2/_3$	-20	50
-1	$6^2/_3$	-50
13.5	$6^2/_3$	30

The results of the calculations are shown in table 13; parameter *a* refers to the own effect, parameter *b* to that of the nearest neighbor, *c* being a (non-significant) constant.

Table n°13 : Computation results.

Parameters	***Values***	***Probability***
a	2.0230	.26
b	-1.5952	.33
c	-5.7767	.75
R^2	.6965	.30

Given the few degrees of freedom, no coefficient is significant; the interesting point is the competitivity of the nearest neighbor. No multiple regime approach has been applied, given the demonstrative nature of the exercise.

The process was simulated, but given its explosive character, the *a* and *b* parameters were divided by *10,* and the constant was deleted; figure 8 presents the graph.

The explosive nature may be due to two factors: the discretization of the process, and the introduction of the spatial potential. To have some insight in this feature, the model was simulated without the potential; figure 9 shows the result.

The process is less divergent at its beginning, but later on becomes explosive, a fact which now could be only due to discretization (remember that the wave equation produces a cyclic pattern).

Figure n°8 : Simulation of the computed PPFDE

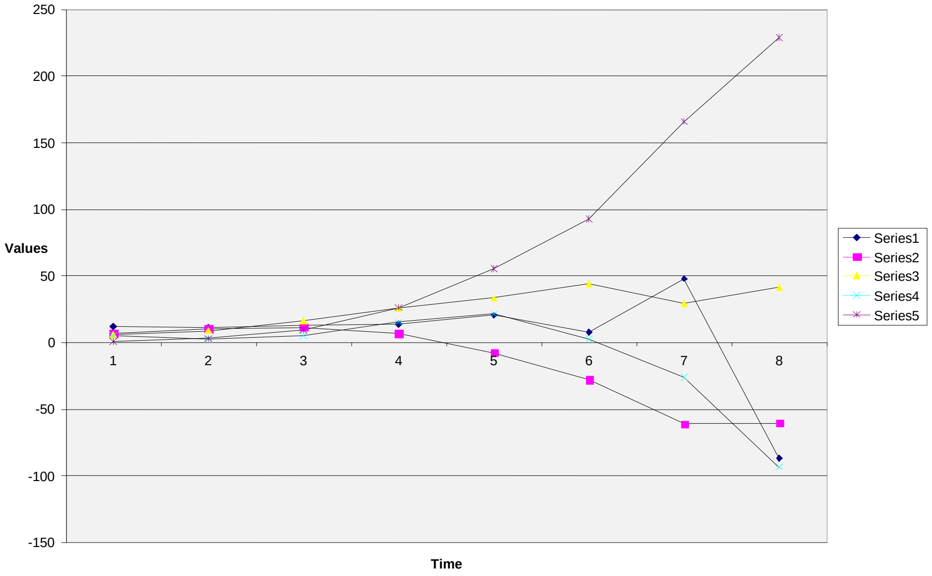
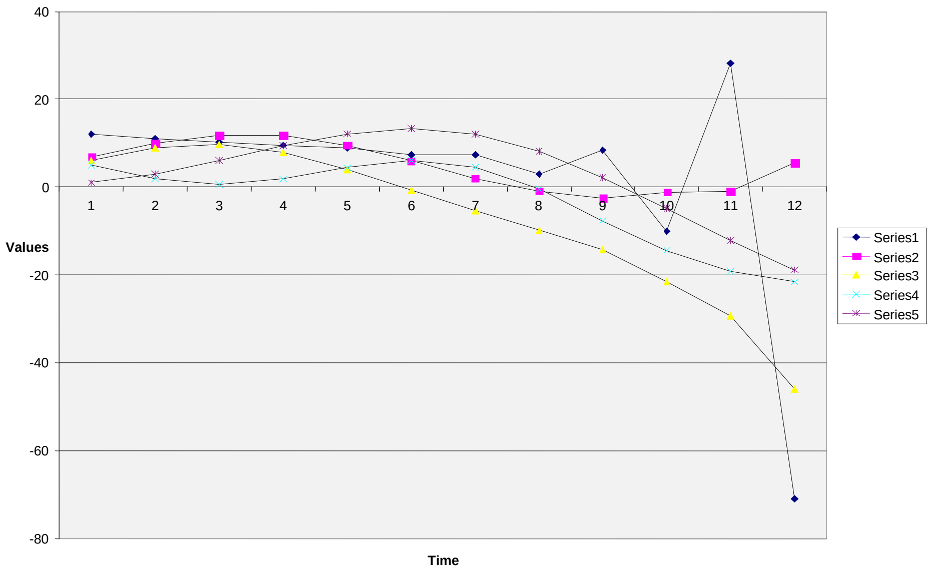
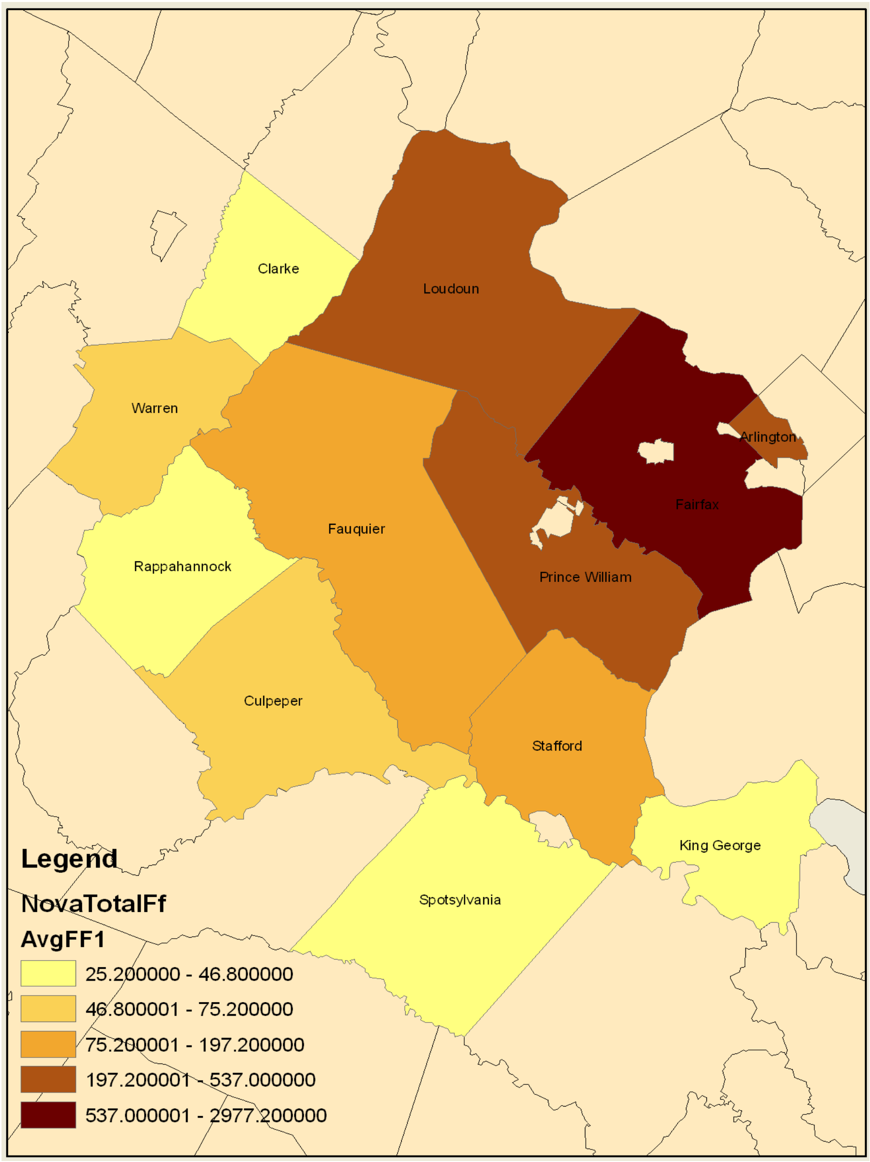


Figure n°9 : Simulation without potential



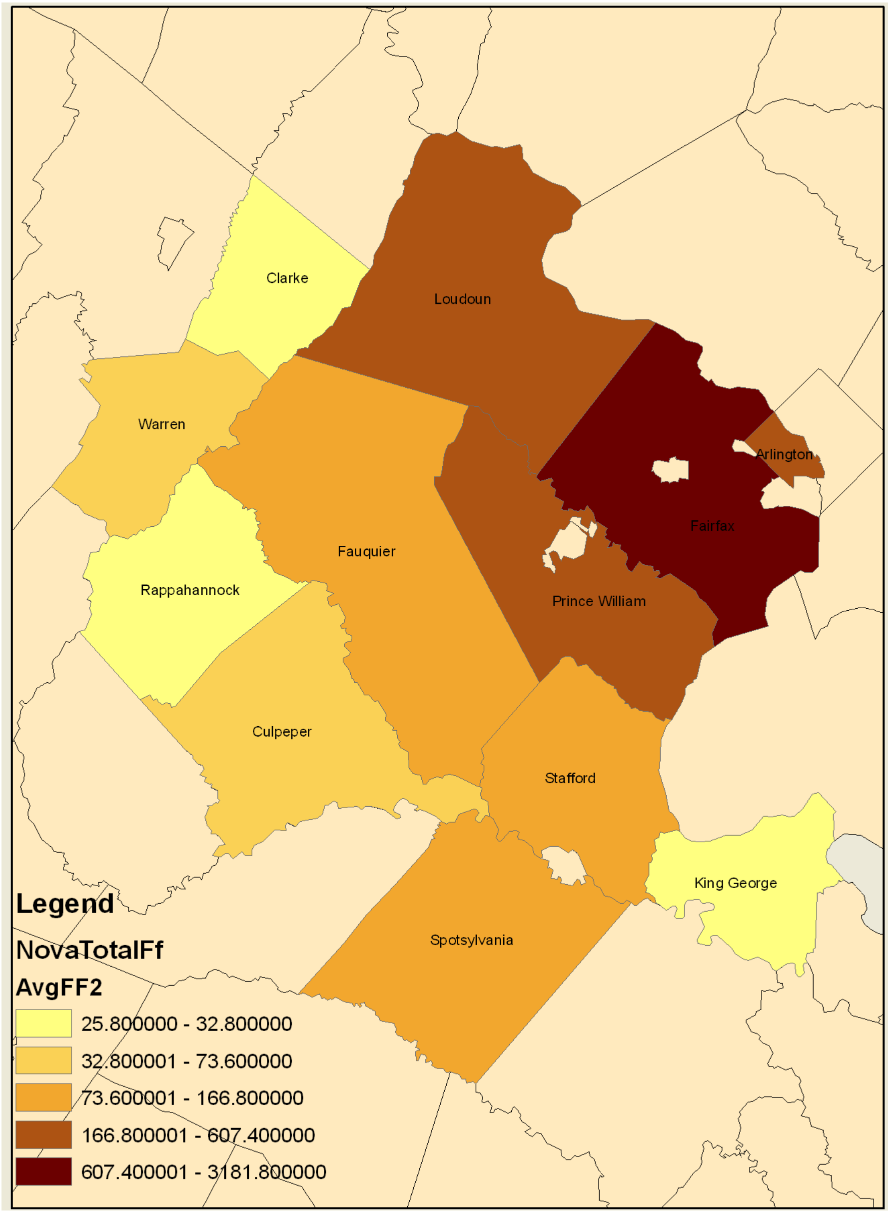
Map n°3 : 1990-1994



The model is now applied to figures from the 12 counties of Northern Virginia; they concern the creation of new enterprises over the period 1990-2003 (figures from the US Bureau of Census). To compute the time second differential it was split in 5-5-4 years, and the overall averages were used to compute the space second differentials. Longitudes and latitudes of main centers and the distances between them (not reproduced here) were also available to compute the Laplace operators. Four "limiting" counties were excluded from the regression analysis below, as no external differences could be computed.

Maps 3 through 5 picture the three period averages.

Map n°4 : 1995-1999



Map n°5 : 2000-2004

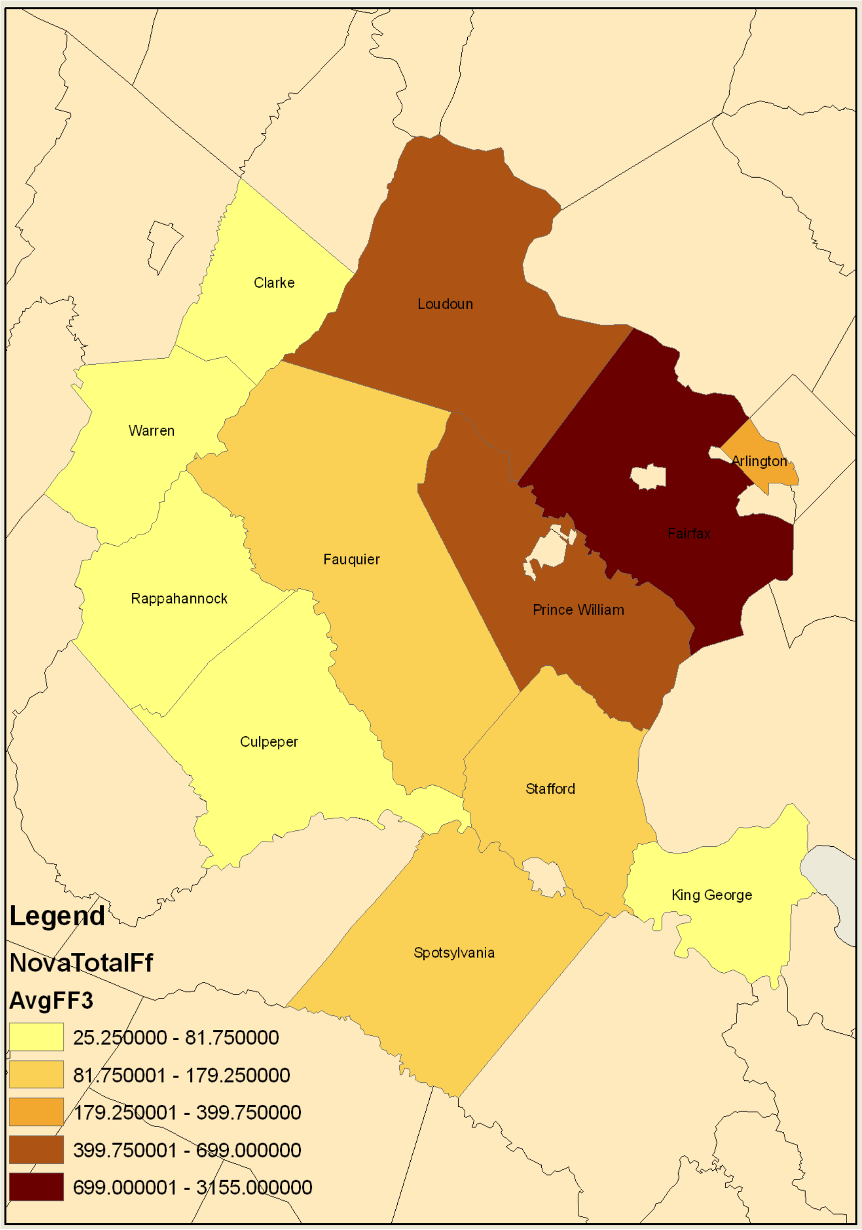


Table 14 reproduces the derived magnitudes, the last column referring again to the nearest neighbor.

Table n°14 : Starting differentials

County	$\Delta^{2\prime}_t$	∇	∇*
Arlington	4.35	.3624	-38.0739
Culpeper	21.75	-.0733	1.7358
Fairfax	-242.60	-.38.0739	12.2183
Fauquier	-11.25	3.0107	12.9183
Loudoun	36.45	8.4519	-38.0739
Prince William	34.45	12.2183	-38.0739
Rappahanock	.85	1.7358	-.0733
Stafford	42.85	.8194	-38.0739

Table 15 reproduces the regression results; as in sections 7 and 8 two regimes were detected, this approach was introduced again. The parameters correspond to those of table 13.

Table n°15 : Regression results

Specification	***Parameters***	***Values***	***Probability***
OLS	a	6.2374	.ooo6
	b	.0444	.42484
	c	-.9286	.9380
Correlation	R^2	.9366	.0010
Regime1	a	7.2543	
	b	.6503	
	c	25.5612	
Regime 2	a	-.3816	
	b	-.9882	
	c	1.7289	
Correlation	R^2	.9995	

Remarkable is again the fact that the coefficients of the two-regimes case have opposite signs. To regime 1 belong Arlington, Culpeper, Fairfax and Stafford; Arlington and Fairfax are Washington DC oriented; but an explanation for the other two counties is still to be found.

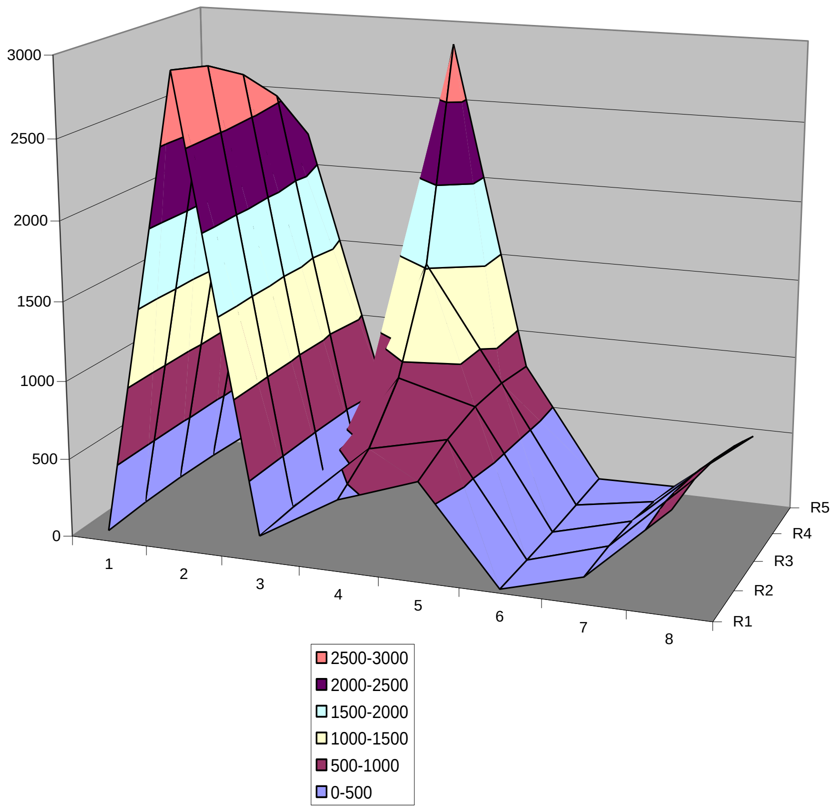
As in simulations some figures could turn negative, logarithms were used to derive new estimates with quite different results as table 16 shows (again two regimes; in the second case constant omitted).

Table n°16 : Logarithmic results

Parameters	*Values 1*	*Values 2*
a	-21.2740	-20.6710
b	4.3350	4.4867
c	-.0285	-
a	.8378	.8332
b	-2.7587	-2.1317
c	-.0296	-
R^2	.9961	.9852

As can be seen from the results, quite some differences appear in comparison with table 14; the parameters a and b show opposite signs, and the regimes apply to different counties (in case 1 to Culpeper, Fauquier, Loudoun and Stafford, in case 2 only to Culpeper, Loudoun and Stafford).Values 2 have been used for a simulation (figure 10) which again shows a divergent tendency.

Figure n°10 : Simulation results



10. CONCLUSIONS

This study is part of a general series on exploring the possibilities of non-standard dynamic specifications (see e.g. Paelinck, 2003 and 2004b); earlier studies have proved analytical tools to be most powerful in representing multiregional dynamics. As said in section 3, though complex patterns may appear, they are of the nature of controllable "exact chaos".

It seems to be possible to model extremely complex spatial developments by means of relatively simple specifications (Wolfram, 2002), though spatial interaction has to be taken into account.

For the future, possible extensions are envisaged :

- use the full 1990-2003 US series to model the above developments; indeed, estimators may have to differ according to the uses they are put to (testing ; simulation)

-apply the model to the French data.

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ÉQUATIONS AUX DIFFÉRENCES FINIES PARTIELLES POTENTIALISÉES : ANALYSE DE LA COMPLEXITÉ DU DÉVELOPPEMENT ÉCONOMIQUE DANS L'ESPACE D'ACTIVITÉS DE CONNAISSANCE

Résumé - *Les études régionales et urbaines d'activités de connaissance sont nombreuses. Une analyse systématique des différents types de liaisons entre ces activités hautement techniques est donnée au départ de la présente étude. Un outil approprié est défini pour représenter la dynamique de ces liaisons : les équations aux dérivées partielles potentialisées (EDPP), qui permettent de représenter des relations spatio-temporelles dans l'espace pré-géographique. Comme en pratique seulement des données discrètes sont disponibles, il faut spécifier des équations aux différences finies partielles potentialisées (EDFPP). On a traité un cas et simulé ses résultats qui indiquent un degré très élevé de complexité spatio-temporelle. Une estimation économétrique spatiale est possible, comme d'autres travaux l'ont démontré. Elle est appliquée à des données françaises. Enfin, différentes versions ont été investiguées; elles sont présentées avec un exercice sur données américaines.*